

71/13

A
T R E A T I S E
O F
P R A C T I C A L G E O M E T R Y.
I N T H R E E P A R T S.

By the late Dr DAVID GREGORY,

Some time Professor of Mathematics in the University of EDINBURGH, and afterwards *Savilian*
Professor of Astronomy at OXFORD.

[Translated from the Latin. With Additions.]

T H E T E N T H E D I T I O N.

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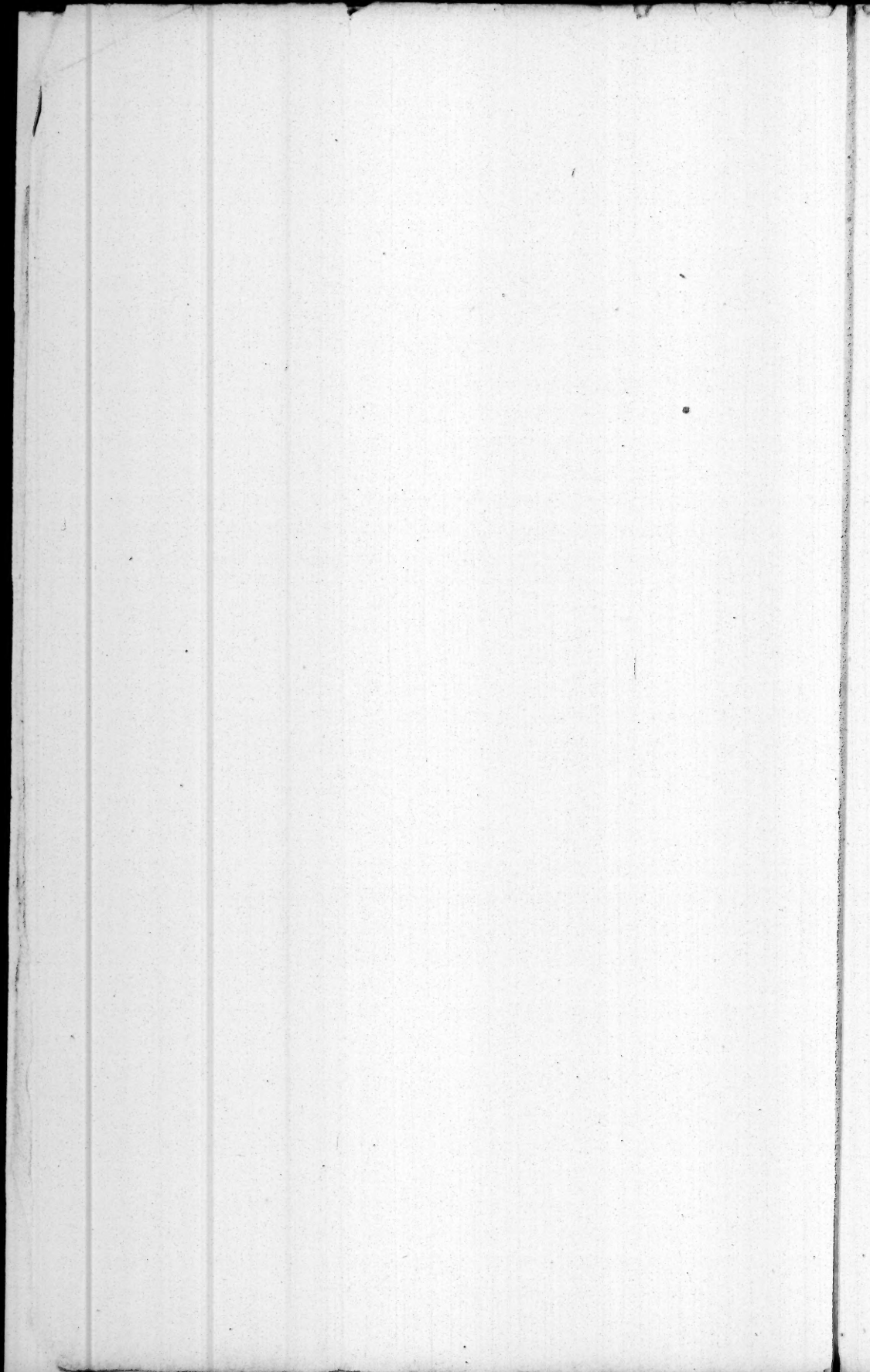


P R E F A C E.

THIS Treatise was compoted in Latin, about sixty years ago, by Dr DAVID GREGORY, then Professor of Mathematics in the Univerfity of Edinburgh; where it has been constantly taught, fince that time, immediately after Euclid's Elements, and the Plain Trigonometry, as proper for exercifing the Students in the Application of Geometry to Practice. The Bookseller having procured an Englifh Tranflation of it, which had been made by an ingenious Gentleman when a Student here, this Tranflation has been revifed, and feveral Additions have been made to the Treatife itfelf, in order to render it more ufeul at this time. The Reader will find thefe diftinguifhed from the Author's Text.

COL. M'LAURIN.

College of Edinburgh,
May 1. 1745.



A
T R E A T I S E
O F
PRACTICAL GEOMETRY.

HAVING explained the first six books of Euclid, with the eleventh and twelfth, which may serve for geometrical elements, and having also taught the Plain Trigonometry, we are now to subjoin some corollaries which are easily deduced from them, that contain practical rules of great use in the affairs of life, concerning the mensuration of lines, angles, surfaces, and solids.

This

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This Treatise of Practical Geometry is divided into three parts. In the first, we treat of the mensuration of lines and angles; to which we have subjoined surveying. In the second, we treat of surfaces; not of such as are plain only, but of some curve surfaces likewise; as of the surface of the cylinder, cone, and sphere; and of those parts of the sphere which we have frequently occasion to consider. It is shewn how to express the area of these in the superficial measures that are now in use amongst us. The third part treats of solid figures and their mensuration. After deducing the rules for finding the solid content of the parallelopipedon, prism, pyramid, cylinder, cone, &c. from Euclid, we add, from Archimedes, the mensuration of the sphere and spheroid, and of their segments, demonstrated in an easy manner; from whence a method is derived for finding the contents of vessels that are either full, or in part empty, in the wet as well as the dry measures, that are now in use amongst us.

PART

P A R T I.

A Line, or length, to be measured, whether it be distance, height, or depth, is measured by a line less than it. With us, the least measure of length is an inch: Not that we measure no line less than it, but because we do not use the name of any measure below that of an inch; expressing lesser measures by the fractions of an inch: And in this Treatise we use decimal fractions as the easiest. Twelve inches make a foot; three feet and an inch make the Scots ell; six ells make a fall; forty falls make a furlong; eight furlongs make a mile: So that the Scots mile is 1184 paces, accounting every pace to be five feet. These things are according to the statutes of Scotland; notwithstanding which, the glaziers use a foot of only eight inches; and other artists, for
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the most part, use an English foot, on account of the several scales marked on the English foot-measure for their use. But the English foot is somewhat less than the Scots; so that 185 of these make 186 of those.

Lines, to the extremities, and any intermediate point, of which you have easy access, are measured by applying to them the common measure a number of times. But lines, to which you cannot have such access, are measured by methods taken from Geometry; the chief whereof we shall here endeavour to explain. The first is by the help of the geometrical square.

“As for the English measures, the yard
“is three feet, or thirty-six inches. A pole
“is sixteen feet and a half, or five yards
“and a half. The chain commonly called
“Gunter’s chain, is four poles, or twenty-
“two yards, that is, sixty-six feet. An
“English statute mile is fourscore chains,
“or 1760 yards, that is, 5280 feet.

“The

“ The chain, (which is now much in
 “ use, because it is very convenient for sur-
 “ veying), is divided into a hundred links,
 “ each of which is $7\frac{9}{100}$ of an inch: Whence
 “ it is easy to reduce any number of those
 “ links to feet, or any number of feet to
 “ links.

“ A chain that may have the same ad-
 “ vantages in surveying in Scotland, as
 “ Gunter’s chain has in England, ought to
 “ be in length seventy-four feet, or twen-
 “ ty-four Scots ells, if no regard is had to
 “ the difference of the Scots and English
 “ foot above mentioned. But, if regard
 “ is had to that difference, the Scots chain
 “ ought to consist of $74\frac{2}{3}$ English feet, or
 “ 74 feet 4 inches and $\frac{4}{3}$ of an inch. This
 “ chain being divided into an hundred
 “ links, each of those links is 8 inches and
 “ $\frac{9}{100}$ of an inch. In the following table,
 “ the most noted measures are expressed
 “ in English inches, and decimals of an
 “ inch.”

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	<i>Inch.</i>	<i>Dec.</i>
The English foot is . . .	12	000
The Paris foot . . .	12	788
The Rhinland foot, measured by Mr Picart	12	362
The Scots foot	12	065
The Amsterdam foot, by Snellius and Picart	11	172
The Dantzick foot, by Hevelius	11	297
The Danish foot, by Mr Picart	12	465
The Swedish foot, by the same	11	692
The Brussels foot, by the same	10	828
The Lyons foot, by Mr Auzout	13	458
The Bononian foot, by Mr Cassini	14	938
The Milan foot, by Mr Auzout	15	631
The Roman palm used by mer- chants, according to the same	9	791
The Roman palm used by archi- tects	8	779
The palm of Naples, according to Mr Auzout	10	314
The English yard	36	000
The English ell	45	000
The Scots ell	37	200
		The

PRACTICAL GEOMETRY. 11

	<i>Inch</i>	<i>Dec.</i>
The Paris aune used by mercers, according to Mr Picart	46	786
The Paris aune used by drapers, according to the same . . .	46	680
The Lyons aune, by Mr Auzout	46	570
The Geneva aune . . .	44	760
The Amsterdam ell . . .	26	800
The Danish ell, by Mr Picart	24	930
The Swedish ell . . .	23	380
The Norway ell . . .	24	510
The Brabant, or Antwerp ell	27	170
The Bruffels ell . . .	27	260
The Burges ell . . .	27	550
The brace of Bononia, according to Auzout	25	200
The brace used by architects in Rome	30	730
The brace used in Rome by mer- chants	34	270
The Florence brace used by mer- chants, according to Picart	22	910
The Florence geographical brace	21	570
The vara of Seville	33	127
The		

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	<i>Inch</i>	<i>Dec.</i>
The vara of Madrid . . .	39	166
The vara of Portugal . .	44	031
The cavedo of Portugal . .	27	354
The ancient Roman foot . .	11	632
The Persian arish, according to Mr Greaves . . .	38	364
The shorter pike of Constanti- nople, according to the same	25	576
Another pike of Constantinople, according to Messrs Mallet and De la Porte. . . .	27	920

PROPOSITION I.

PROBLEM I.

To Describe the Structure of the Geometrical Square.

THE Geometrical Square is made of any solid matter, as brass or wood, or of any four plain rulers joined together at

at right angles, (as in fig. 1.); where A is the centre, from which hangs a thread with a small weight at the end, so as to be directed always to the centre. Each of the sides BE and DE is divided into an hundred equal parts, or (if the sides be long enough to admit of it) into a thousand parts; C and F are two sights, fixed on the side AD. There is moreover an index GH, which, when there is occasion, is joined to the centre A, in such manner as that it can move round, and remain in any given situation. On this index are two sights perpendicular to the right line going from the centre of the instrument: These are K and L. The side DE of the instrument is called the upright side; E the reclining side.

PROP.

PROP. II. FIG. 2.

To measure an Accessible Height, AB, by the help of a Geometrical Square, its distance being known.

LET BR be an horizontal plane, on which there stands perpendicularly any line AB: Let BD, the given distance of the observator from the height, be 96 feet; let the height of the observator's eye be supposed 6 feet; and let the instrument, held by a steady hand, or rather leaning on a support, be directed towards the summit A, so that one eye (the other being shut) may see it clearly through the sights; the perpendicular or plum-line mean while hanging free, and touching the surface of the instrument: Let now the perpendicular be supposed to cut off on the right side KN 80 equal parts. It is clear that LKN, ACK, are similar triangles; for the angles LKN, ACK, are right angles, and therefore equal: Moreover,

over, LN and AC are parallel, as being both perpendicular to the horizon ; consequently, by Prop. 29. B. 1. of Euclid, the angles KLN, KAC, are equal ; wherefore, by the second corollary and of the 32. Prop. B 1. of Euclid, the angles LNK and AKC, are likewise equal : So that, in the triangles NKL, KAC, (by the 4. Prop. of the 6. B. of Euclid) as $NK : KL :: KC (i. e. BD) : CA$; that is, as 80 to 100, so is 96 feet to CA. Therefore, by the rule of three, CA will be found to be 120 feet ; and CB, which is 6 feet, being added, the whole height is 126 feet.

But, if the distance of the observer from the height, as BE, be such, that, when the instrument is directed, as formerly, toward the summit A, the perpendicular falls on the angle P, opposite to H, the centre of the instrument, and BE or CG be given of 120 feet ; CA will also be 120 feet. For, in the triangles HGP, ACG, equiangular, as in the preceding case, as $PG : GH :: GC : CA$. But PG is equal to GH ; therefore GC is
likewise

likewise equal to CA: That is, CA will be 120 feet, and the whole height 126 feet, as before.

Let the distance BF be 300 feet, and the perpendicular or plum-line cut off 40 equal parts from the reclining side: Now, in this case, the angles QAC, QZI, are equal, by the 29. Prop. 1. B. of Euclid. And, by the same Prop. the angles QZI, ZIS are equal; therefore the angle ZIS is equal to the angle QAC. But the angles ZSI, QCA are equal, being right angles; therefore in the equiangular triangles ACQ, SZI, by the 4. Prop. of the 6. B. of Euclid, it will be, as $ZS : SI :: CQ : CA$; that is, as 100 to 40, so is 300 to CA. Wherefore, by the rule of three, CA will be found to be of 120 feet. And, by adding the height of the observator, the whole BA will be 126 feet. Note, That the height is greater than the distance, when the perpendicular cuts the right side, and less, if it cut the reclined side; and that the height and distance are equal, if the perpendicular fall on the opposite angle.

SCHO-

SCHOLIUM. FIG. 3.

If the height of a tower to be measured, as above, end in a point, as in Fig. 3. the distance of the observer opposite to it is not CD, but is to be accounted from the perpendicular to the point A; that is, to CD must be added the half of the thickness of the tower, viz. BD: Which must likewise be understood in the following propositions, when the case is similar.

P R O P. III.

PROB. FIG. 4.

From the height of a Tower AB given, to find a distance on the horizontal plane BC, by the Geometrical Square.

LET the instrument be so placed, as that the mark C in the opposite plane may be seen through the sights; and let it

be observed how many parts are cut off by the perpendicular. Now, by what hath been already demonstrated, the triangles AEF, ABC are similar; therefore, by 4th, 6. Eucl. it will be as EF to AE, so AB (composed of the height of the tower BG, and of the height of the centre of the instrument A, above the tower BG) to the distance BC. Wherefore, if, by the rule of three, you say, as EF to AE, so is AB to BC, it will be the distance sought.

P R O P. IV. FIG. 5.

To measure any distance at land or sea by the Geometrical Square.

IN this operation, the index is to be applied to the instrument, as was shown in the description; and, by the help of a support, the instrument is to be placed horizontally at the point A; then let it be turned till the remote point F, whose distance is to be

be measured, be seen through the fixed sights; and bring the index to be parallel with the other side of the instrument; observe by the sights upon it any accessible mark B, at a sensible distance: Then carrying the instrument to the point B, let the immoveable sights be directed to the first station A, and the sights of the index to the point F. If the index cut the right side of the square, as in K, in the two triangles BRK, and BAF, which are equiangular, it will be (by 4th, 6. Eucl.) as BR to RK, so BA (the distance of the stations to be measured with a chain) to AF; and the distance AF sought will be found by the rule of three. But, if the index cut the reclined side of the square in any point L, where the distance of a more remote point is sought; in the triangles BLS, BAG, the side LS shall be to SB, as BA to AG, the distance sought; which accordingly will be found by the rule of three.

PROP.

P R O P. V.

PROB. FIG. 6.

To measure an Accessible Height by means of a plain Mirror.

LET AB be the height to be measured; let the mirror be placed at C, in the horizontal plane BD, at a known distance BC; let the observer go back to D, till he see the image of the summit in the mirror, at a certain point of it, which he must diligently mark; and let DE be the height of the observer's eye. The triangles ABC and EDC are equiangular; for the angles at D and B are right angles; and ACB, ECD are equal, being the angles of incidence and reflection of the ray AC, as is demonstrated in optics; wherefore the remaining angles at A and E are also equal: Therefore, by 4th, 6. Eucl. it will be, as CD to DE, so CB to BA; that this, as the distance

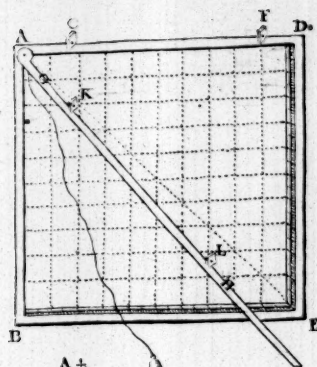


FIG. 1.

FIG. 2.

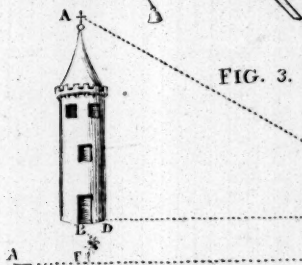
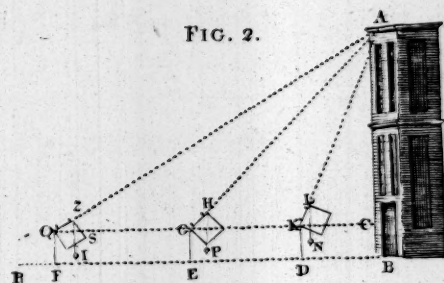


FIG. 3.

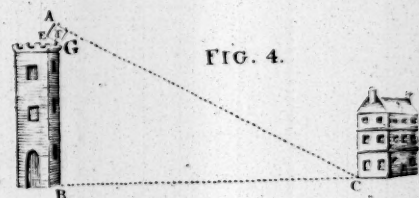


FIG. 4.

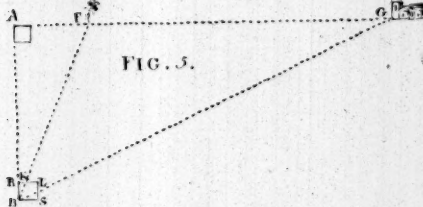


FIG. 5.

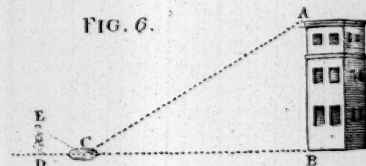


FIG. 6.

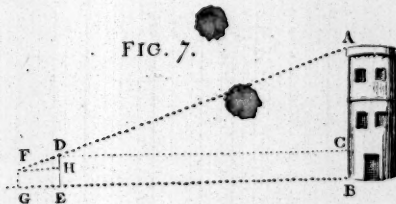


FIG. 7.

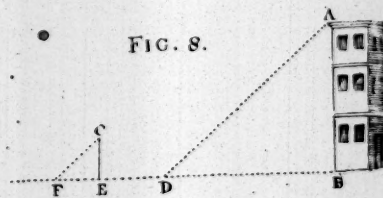


FIG. 8.

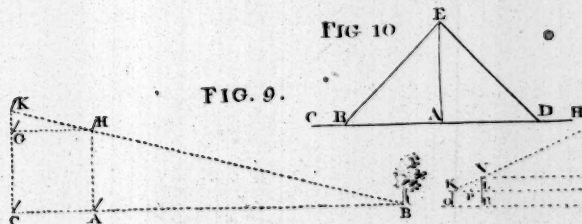


FIG. 9.



FIG. 10.



FIG. PROP. 7.

distance of the observator from the point of the mirror in the right line betwixt the observator and the height, is to the height of the observator's eye, so is the distance of the tower from that point of the mirror, to the height of the tower sought; which therefore will be found by the rule of three.

Note 1. The observator will be more exact, if, at the point D, a staff be placed in the ground perpendicularly, over the top of which the observator may see a point of the glass exactly in a line betwixt him and the tower.

Note 2. In place of a mirror, may be used the surface of water contained in a vessel, which naturally becomes parallel to the horizon.

P R O P.

P R O P. VI. FIG. 7.

To measure an Accessible Height AB by means of two staffs.

LET there be placed perpendicularly in the ground a longer staff DE, likewise a shorter one FG, so as the observator may see A, the top of the height to be measured, over the ends D, F, of the two staffs; let FH and DC, parallel to the horizon, meet DE and AB in H and C; then the triangles FHD, DCA, shall be equiangular; for the angles at C and H are right ones: Likewise the angle A is equal to the angle FDH, by 29th, 1. Eucl.; wherefore the remaining angles DFH, and ADC, are also equal: Wherefore, by 4th, 6. Eucl. as FH, the distance of the staffs, to HD, the excess of the longer staff above the shorter; so is DC, the distance of the longer staff from the tower, to CA, the excess of the height
of

of the tower above the longer staff. And thence CA will be found by the rule of three.

To which, if the length DE be added, you will have the whole height of the tower BA, Q. E. F. •

SCHOLIUM. FIG. 8.

Many other methods may be occasionally contrived for measuring an accessible height. For example, from the given length of the shadow BD, I find out the height AB, thus: Let there be erected a staff CE perpendicularly, producing the shadow EF: The triangles ABD, CEF, are equiangular; for the angles at B, and E, are right; and the angles ADB, and CFE, are equal, each being equal to the angles of the sun's elevation above the horizon: Therefore, by 4th, 6. Eucl. as EF, the shadow of the staff, to EC the staff itself, so BD, the shadow of the tower, to BA, the height of the tower. Though the plane on which the shadow of the

the tower falls be not parallel to the horizon, if the staff be erected in the same plane, the rule will be the same.

P R O P. VII.

To measure an inaccessible Height by means of two staffs.

HITHERTO we have supposed the height to be accessible, or that we can come at the lower end of it; now, if, because of some impediment, we cannot get to a tower, or if the point whose height is to be found out be the summit of a hill, so that the perpendicular be hid within the hill; if, I say, for want of better instruments, such an inaccessible height is to be measured by means of two staffs, let the first observation be made with the staffs DE and FG, as in Prop. VI.; then the observator is to go off in a direct line from the height and first station, till he come to the second station; where he is to place

place the longer staff perpendicularly at RN, and the shorter staff at KO, so that the summit A may be seen along their tops; that is, so that the points K, N, A may be in the same right line. Through the point N let there be drawn the right line NP parallel to FA: Wherefore, in the triangles KNP, KAF, the angles KNP, KAF are equal by the 29. 1. Eucl. also the angle AKF is common to both; consequently the remaining angle KPN is equal to the remaining angle KFA. And therefore, by 4th, 6. Eucl. $PN : FA :: KP : KF$. But the triangles PNL, FAS are similar; therefore, by 4th, 6. Eucl. $PN : FA :: NL : SA$. Therefore, by the 11th, 5. Eucl. $KP : KF :: NL : SA$. Thence, alternately, it will be, as KP (the excess of the greater distance of the short staff from the long one above its lesser distance from it) to NL, the excess of the longer staff above the shorter; so KF, the distance of the two stations of the shorter staff, to SA, the excess of the height sought above the height of the shorter staff. Wherefore SA will be found by

D

the

the rule of three. To which let the height of the shorter staff be added, and the sum will give the whole inaccessible height BA. Q. E. F.

Note 1. In the same manner may an inaccessible height be found by a geometrical square, or by a plain speculum. But we shall leave the rules to be found out by the student, for his own exercise.

Note 2. That by the height of the staff we understand its height above the ground in which it is fixed.

Note 3. Hence depends the method of using other instruments invented by geometricians; for example, of the geometrical crosses: And, if all things be justly weighed, a like rule will serve for it as here. But we incline to touch only upon what is most material.

P R O P.

PROP. VIII. FIG. 9.

To measure the distance AB, to one of whose extremities we have access, by the help of four staffs.

LET there be a staff fixed at the point A; then going back at some sensible distance in the same right line, let another be fixed in C, so as that both the points A and B be covered and hid by the staff C; likewise going off in a perpendicular from the right line CB, at the point A, (the method of doing which shall be shown in the following *scholium*), let there be placed another staff at H; and in the right line CKG, (perpendicular to the same CB, at the point B), and at the point of it K, such that the points K, H, and B, may be in the same right line, let there be fixed a fourth staff. Let there be drawn, or let there be supposed to be drawn, a right line HG parallel to CA. The triangles KGH, HAB, will be equiangular;

angular; for the angles HAB, KGH are right angles. Also, by 29th, 1. Eucl. the angles ABH, KHG are equal; wherefore, by the 4th, 6. Eucl. as KG (the excess of CK above AH) to GH, or to CA, the distance betwixt the first and second staff; so is AH, the distance betwixt the first and third staff, to AB the distance sought.

SCHOLIUM. FIG. 10.

To draw on a plane a right line AE perpendicular to CH, from a given point A; take the right lines AB, AD, on each side equal; and in the points B and D, let there be fixed stakes, to which let there be tied two equal ropes BE, DE, or one having a mark in the middle, and holding in your hand their extremities joined, (or the mark in the middle, if it be but one), draw out the ropes on the ground; and then, where the two ropes meet, or at the mark, when by it the rope is fully stretched, let there be placed

placed a third stake at E ; the right line AE will be perpendicular to CH in the point A, by 11th, 1. Eucl. In a manner not unlike to this, may any problems, that are resolved by the square and compasses, be done by ropes and a cord turned round as a radius.

P R O P. IX. FIG. 11.

To measure the distance AB, one of whose extremities is accessible.

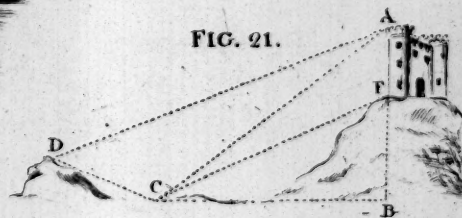
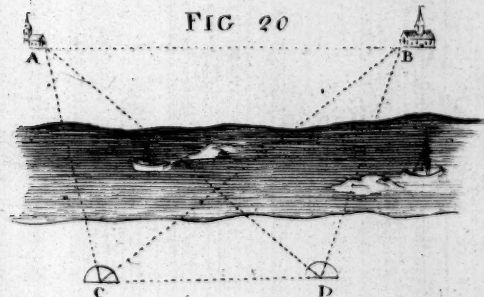
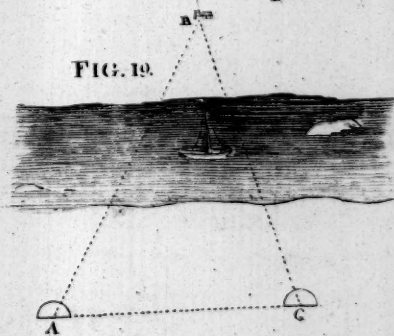
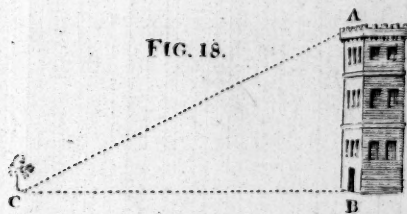
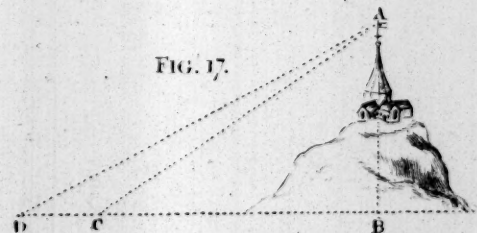
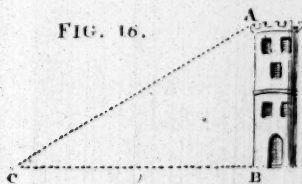
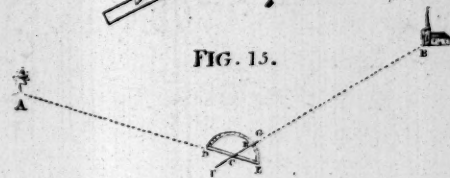
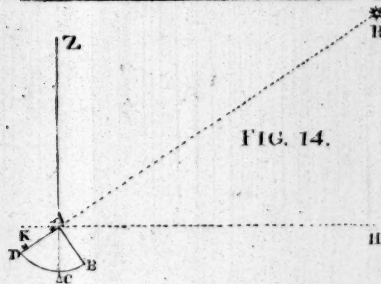
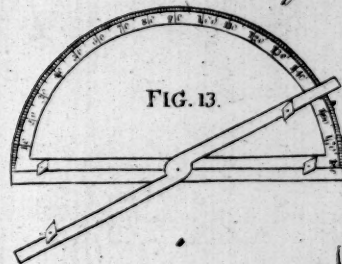
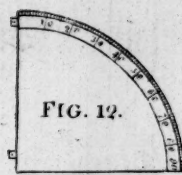
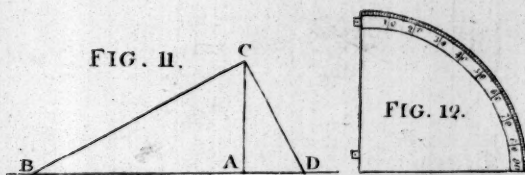
FR O M the point A, let the right line AC, of a known length, be made perpendicular to AB, (by the preceding *scho- lium*): Likewise draw the right line CD perpendicular to CB, meeting the right line AB in D: Then, by the 8th, 6. Eucl. as $DA : AC :: AC : AB$. Wherefore, when DA and AC are given, AB will be found by the rule of three. Q. E. F.

S C H O L I U M.

All the preceding operations depend on the equality of some angles of triangles,
and

and on the similitude of the triangles arising from that equality. And on the same principles depend innumerable other operations, which a geometrician will find out of himself, as is very obvious. However, some of these operations require such exactness in the work, and without it are so liable to errors, that, *caeteris paribus*, the following operations, which are performed by a trigonometrical calculation, are to be preferred; yet could we not omit those above, being most easy in practice, and most clear and evident to those who have only the first elements of geometry. But, if you are provided with instruments, the following operations are more to be relied upon. We do not insist on the easiest cases to those who are skilled in plain trigonometry, which is indeed necessary to any one who would apply himself to practice. It would be easy to the reader to find examples; and we have shown, in plain trigonometry, how to find the angle or side of any plain triangle that is required, from the angles or sides that may be given.

P R O P.



PROP. X. FIG. 12.

To describe the Construction and Use of the Geometrical Quadrant.

THE Geometrical Quadrant is the fourth part of a circle divided into ninety degrees, to which two sights are adapted, with a perpendicular or plumb line hanging from the centre. The general use of it is for investigating angles in a vertical plane, comprehended under right lines going from the centre of the instrument, one of which is horizontal, and the other is directed to some visible point. This instrument is made of any solid matter, as wood, copper, &c.

PROP.

PROP. XI. FIG. 13.

To describe the Make and Use of the Graphometer.

THE Graphometer is a semicircle made of any hard matter, of wood, for example, or brass, divided into 180 degrees; so fixed on a *fulcrum*, by means of a brass ball and socket, that it easily turns about, and retains any situation; two sights are fixed on its diameter. At the centre there is commonly a magnetical needle in a box. There is likewise a moveable ruler, which turns round the centre, and retains any situation given in it. The use of it is to observe any angle, whose vertex is at the centre of the instrument in any plane, (though it is most commonly horizontal, or nearly so), and to find how many degrees it contains.

PROP.

P R O P. XII.

FIG. 14. and 15.

To describe the manner in which Angles are measured by a Quadrant or Graphometer.

LET there be an angle in a vertical plane, comprehended between a line parallel to the horizon HK, and the right line RA, coming from any remarkable point of a tower or hill, or from the sun, moon, or a star. Suppose that this angle RAH is to be measured by the quadrant: Let the instrument be placed in the vertical plane, so as that the centre A may be in the angular point: And let the sights be directed towards the object at R, (by the help of the ray coming from it, if it be the sun or moon, or by the help of the visual ray, if it is any thing else), the degrees and minutes in the

E arch

arch BC, cut off by the perpendicular, will measure the angle RAH required. For, from the make of the quadrant, BAD is a right angle; therefore BAR is likewise right, being equal to it. But, because HK is horizontal, and AC perpendicular, HAC will be a right angle; and therefore equal also to BAR. From those angles subtract the part HAB that is common to both; and there will remain the angle BAC equal to the angle RAH. But the arch BC is the measure of the angle BAC; consequently it is likewise the measure of the angle RAH.

Note, That the remaining arch on the quadrant DC is the measure of the angle RAZ, comprehended between the foresaid right line RA and AZ, which points to the zenith.

Let it now be required to measure the angle ACB (Fig. 15.) in any plane, comprehended between the right lines AC and BC, drawn from two points A and B, to the place of station C. Let the graphometer be placed at C, supported by its *fulcrum* (as was shown above); and let the immoveable
 fights

sights on the side of the instrument DE be directed towards the point A; and likewise (while the instrument remains immoveable) let the sights of the ruler FG (which is moveable about the center C) be directed to the point B. It is evident that the moveable ruler cuts off an arch DH, which is the measure of the angle ACB sought. Moreover, by the same method, the inclination of CE, or of FG, may be observed with the meridian line, which is pointed out by the magnetic needle inclosed in the box, and is moveable about the center of the instrument, and the measure of this inclination or angle found in degrees.

PROP. XIII. FIG. 16.

To measure an Accessible Height by the Geometrical Quadrant.

BY the 12th Prop. of this part, let the angle C be found by means of the quadrant. Then in the triangle ABC, right-angled

angled at B, (BC being supposed the horizontal distance of the observer from the tower), having the angle at C, and the side BC, the required height BA will be found by the 3d case of plain trigonometry.

PROP. XIV. FIG. 17.

To measure an Inaccessible Height by the Geometrical Quadrant.

LET the angle ACB be observed with the quadrant (by the 12th Prop. of this part;) then let the observer go from C to the second station D, in the right line BCD, (providing BCD be a horizontal plane;) and, after measuring this distance CD, take the angle ADC likewise with the quadrant. Then, in the triangle ACD, there is given the angle ADC, with the angle ACD; because ACB was given before: Therefore (by 32. 1. Eucl.) the remaining angle CAD is given likewise. But the side CD is likewise

wise given, being the distance of the station C and D; therefore (by the first case of oblique angled triangles in Trigonometry) the side AC will be found. Wherefore, in the right-angled triangle ABC, all the angles and the hypotheneuse AC are given; consequently, by the 4th case of Trigonometry, the height sought AB will be found; as also (if you please) the distance of the station C from AB, the perpendicular within the hill or inaccessible height.

PROP. XV. FIG. 18.

From the top of a given height, to measure the distance BC.

LET the angle BAC be observed by the 12th of this part; wherefore, in the triangle ABC, right-angled at B, there is given, by observation, the angle at A; whence (by the 32d, 1. Eucl.) there will also be given the angle BCA. Moreover,
the

38 A TREATISE OF

the side AB (being the height of the tower) is supposed to be given. Wherefore, by the 3d case of Trigonometry, BC, the distance sought, will be found.

P R O P. XVI. FIG. 19.

To measure the distance of two places A and B, of which one is accessible, by the Graphometer.

LET there be erected at two points A and C, sufficiently distant, two visible signs; then (by the 12th of this) let the two angles BAC, BCA, be taken by the Graphometer. Let the distance of the stations A and C be measured with a chain. Then the third angle B being known, and the side AC being likewise known; therefore, by the first case of Trigonometry, the distance required, AB, will be found.

P R O P.

PROP. XVII. FIG. 20.

To measure by the Graphometer, the distance of two places, neither of which is accessible.

LET two stations, C and D, be chosen, from each of which the places may be seen whose distance is sought: Let the angles ACD, ACB, BCD, and likewise the angles BDC, BDA, CDA, be measured by the Graphometer; let the distance of the stations C and D be measured by a chain, or (if it be necessary) by the preceding practice. Now, in the triangle ACD, there are given two angles ACD and ADC; therefore the third, CAD, is likewise given. Moreover, the side CD is given; therefore, by the first case of Trigonometry, the side AD will be found. After the same manner, in the triangle BCD, from all the angles and one side CD given, the side BD is found. Wherefore, in the triangle ADB, from the given sides DA and DB, and the angle ADB contained

contained by them, the side AB (the distance sought) is found by the fourth case of Trigonometry of oblique-angled triangles.

Let it be noted, that it is not necessary that the points A, B, C, and D, be in one plane; and that any triangle is in one plane, by 2d Prop. 11th of Eucl.

PROP. XVIII. FIG. 21.

It is required by the Graphometer and Quadrant, to measure an Accessible Height AB, placed so on a steep, that one can neither go near it, in an horizontal plane, nor recede from it, as we suppose in the solution of the 14th Prop.

LET there be chosen any situation, as C, and another, D; where let some mark be erected: Let the angles ACD and ADC be found by the Graphometer; then the third angle DAC will be known. Let the side CD, the distance of the stations, be measured

measured with a chain, and thence (by Trigon.) the side AC will be found. Again, in the triangle ACB, right-angled at B, having found by the Quadrant the angle ACB, the other angle CAB is known likewise: But the side AC in the triangle ADC is already known; therefore the height required, AB, will be found by the 4th case of right-angled triangles. If the height of the tower is wanted, the angle BCF will be found by the Quadrant; which being taken from the angle ACB, already known, the angle ACF will remain: But the angle FAC was known before; therefore the remaining angle AFC will be known. But the side AC was also known before; therefore, in the triangle AFC, all the angles, and one of the sides, AC, being known, AF, the height of the tower above the hill, will be found by Trigonometry.

F

SCHO-

S C H O L I U M.

It were easy to add many other methods of measuring heights and distances; but, if what is above be understood, it will be easy (especially for one that is versed in the elements) to contrive methods for this purpose, according to the occasion: So that there is no need of adding any more of this sort. We shall subjoin here a method by which the diameter of the earth may be found out.

P R O P. XIX. FIG. 22.

To find the Diameter of the Earth from one observation.

LET there be chosen a high hill AB, near the sea-shore, and let the observer on the top of it, with an exact Quadrant, divided into minutes and seconds by transverse divisions, and fitted with a telescope,

scope, in place of the common sights, measure the angle ABE contained under the right line AB, which goes to the centre, and the right line BE, drawn to the sea, a tangent to the globe at E; let there be drawn from A, perpendicular to BD, the line AF meeting BE in F. Now, in the right-angled triangle, BAF, all the angles are given, also the side AB, the height of the hill; which is to be found by some of the foregoing methods, as exactly as possible; and, by Trigonometry, the sides BF and AF are found. But, by Corol. 36. 3. Eucl. AF is equal to FE; therefore BE will be known. Moreover, by 36th, 3. Eucl. the rectangle under BA and BD is equal to the square of BE. And thence, by 17th, 6. Eucl. as $AB : BE :: BE : BD$. Therefore, since AB and BE are already given, BD will be found by 11th, 6. Eucl. or by the rule of three; and, subtracting BA, there will remain AD, the diameter of the earth sought.

S C H O-

Many other methods might be proposed for measuring the diameter of the earth. The most exact, in my opinion, is that proposed by Mr *Picart*, of the Academy of Sciences at *Paris*. But, since it does not belong to this place, we refer you to the Philosophical Transactions, where you will find it described.

“ According to Mr *Picart*, a degree of
“ the meridian at the latitude of $49^{\circ} 21'$,
“ was 57,060 *French Toises*, each of which
“ contains six feet of the same measure;
“ from which it follows, that, if the earth
“ be an exact sphere, the circumference of
“ a great circle of it will be 123,249,600
“ *Paris* feet, and the semidiameter of the
“ earth, 19,615,800 feet. But the *French*
“ Mathematicians, who of late have examined Mr *Picart*'s operations, assure us,
“ That the degree in that latitude is 57,183
“ *Toises*. They measured a degree in *Lap-*
“ *land*, in the latitude of $66^{\circ} 20'$, and found
“ it

“ it of 57,438 *Toises*. By comparing these
 “ degrees, as well as by the observations on
 “ pendulums, and the theory of gravity, it
 “ appears that the earth is an oblate spheroid ; and (supposing those degrees to be
 “ accurately measured) the axis or diameter that passes through the poles will be
 “ to the diameter of the equator as 177 to
 “ 178, or the earth will be 22 miles higher
 “ at the equator than at the poles. A degree has likewise been measured at the
 “ equator, and found to be considerably
 “ less than at the latitude of *Paris* ; which
 “ confirms the oblate figure of the earth.
 “ But an account of this last mensuration
 “ has not been published as yet. If the
 “ earth was of an uniform density from the
 “ surface to the centre, then, according to
 “ the theory of gravity, the meridian would
 “ be an exact ellipsis, and the axis would
 “ be to the diameter of the equator as 230
 “ to 231 ; and the difference of the semidiameter of the equator and semiaxis about
 “ 17 miles.”

In

In what follows, a figure is often to be laid down on paper, like to another figure given; and because this likeness consists in the equality of their angles, and in the sides having the same proportion to each other, (by the definitions of the 6th of Eucl.) we are now to shew what methods practical Geometricians use for making on paper an angle equal to a given angle, and how they constitute the sides in the same proportion. For this purpose they make use of a Protractor, (or, when it is wanting, a Line of chords), and of a Line of equal parts.

P R O P. XX.

FIG. 23. 24. 25. 26. and 27.

To describe the Construction and Use of the Protractor, of the Line of Chords, and of the Line of Equal Parts.

THE Protractor is a small semicircle of brass, or such solid matter. The semicircumference is divided into 180 degrees.
The

The use of it is, to draw angles on any plane, as on paper, or to examine the extent of angles already laid down. For this last purpose, let the small point in the centre of the protractor be placed above the angular point, and let the side AB coincide with one of the sides that contain the angle proposed; the number of degrees cut off by the other side, computing on the Protractor from B, will show the quantity of the angle that is to be measured.

But, if an angle is to be made of a given quantity on a given line, and at a given point of that line, let AB coincide with the given line, and let the centre A of the instrument be applied to that point. Then let there be a mark made at the given number of degrees; and a right line drawn from that mark to the given point, will constitute an angle with the given right line, of the quantity required; as is manifest.

This is the most natural and easy method, either for examining the extent of an
angle

angle on paper, or for describing on paper an angle of a given quantity.

But, when there is scarcity of instruments, or because a line of chords is more easily carried about; (being described on a ruler on which there are many other lines besides), practical Geometricians frequently make use of it. It is made thus: Let the quadrant of a circle be divided into 90 degrees, (as in Fig. 24.) The line AB is the chord of 90 degrees; the chord of every arch of the quadrant is transferred to this line AB, which is always marked with the number of degrees in the corresponding arch.

Note, That the chord of 60 degrees is equal to the radius, by Corol. 15th 4. Eucl. If now a given angle EDF is to be measured by the line of chords, from the centre D, with the distance DG, (the chord of 60 degrees), describe the arch GF; and let the points G and F be marked where this arch intersects the sides of the angle. Then, if the distance GF, applied on the line of chords

chords from A to B, gives (for example) 25 degrees, this shall be the measure of the angle proposed.

When an obtuse angle is to be measured with this line, let its complement to a semicircle be measured, and thence it will be known. It were easy to transfer to the diameter of a circle the chords of all arches to the extent of a semicircle; but such are rarely found marked upon rules.

But now, if an angle of a given quantity, suppose of 50 degrees, is to be made at a given point M of the right line KL (Fig. 26.) from the centre M, and the distance MN, equal to the chord of 60 degrees, describe the arch QN. Take off an arch NR, whose chord is equal to that of 50 degrees on the line of chords; join the points M and R; and it is plain that MR shall contain an angle of 50 degrees with the line KL proposed.

But sometimes we cannot produce the sides, till they be of the length of a chord of 60 degrees on our scale; in which case

G

it

it is fit to work by a circle of proportions, (that is a Sector), by which an arch may be made of a given number of degrees to any radius.

The quantities of angles are likewise determined by other lines usually marked upon rules, as the lines of sines, tangents, and secants; but, as these methods are not so easy or so proper in this place, we omit them.

To delineate figures similar or like to others given, besides the equality of the angles, the same proportion is to be preserved among the sides of the figure that is to be delineated, as is among the sides of the figures given. For which purpose, on the rules used by artists, there is a line divided into equal parts, more or less in number, and greater or lesser in quantity, according to the pleasure of the maker.

A foot is divided into inches; and an inch, by means of transverse lines, into 100 equal parts; so that, with this scale, any number of inches, below twelve, with any
part

part of an inch, can be taken by the compasses, providing such part be greater than the one hundredth part of an inch. And this exactness is very necessary in delineating the plans of houses, and in other cases.

P R O P. XXI. FIG. 28.

To lay down on paper, by the Protractor or line of chords, and line of equal parts, a right-lined figure like to one given, providing the angles and sides of the figure given be known by observation or mensuration.

FOR example, suppose that it is known that, in a quadrangular figure, one side is of 235 feet, that the angle contained by it and the second side is of 84° , the second side of 288 feet, the angle contained by it and the third side of 72° , and that the third side is 294 feet. These things being given, a figure is to be drawn on
paper

paper like to this quadrangular figure. On your paper, at a proper point A, let a right line be drawn, upon which take 235 equal parts, as AB. The part representing a foot is taken greater or lesser, according as you would have your figure greater or less. In the adjoining figure, the 100th part of an inch is taken for a foot. And, accordingly, an inch divided into 100 parts, and annexed to the figure, is called a scale of 100 feet. Let there be made at the point B (by the preceding Prop.) an angle ABC of 85° , and let BC be taken of 288 parts like to the former. Then let the angle BCD be made of 72° , and the side CD of 294 equal parts. Then let the side AD be drawn; and it will complete the figure like to the figure given. The measures of the angle A and D can be known by the protractor or line of chords, and the side AD by the line of equal parts; which will exactly answer to the corresponding angles, and to the side of the primary figure.

After

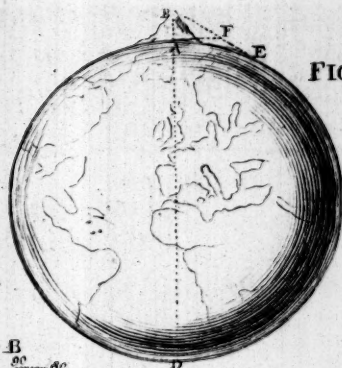


FIG. 22.

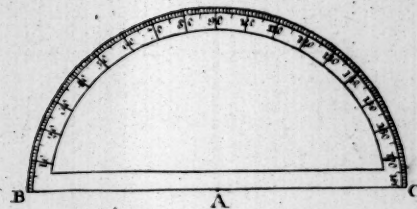


FIG. 23.

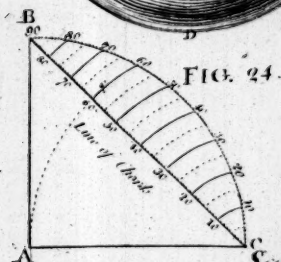


FIG. 24.

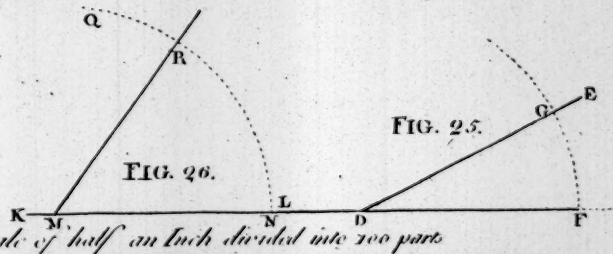


FIG. 25.



Scale of half an Inch divided into 100 parts

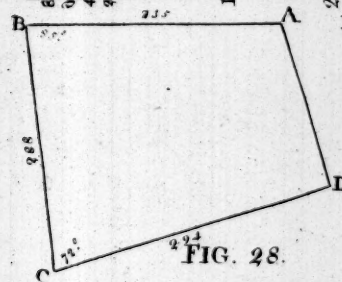


FIG. 28.

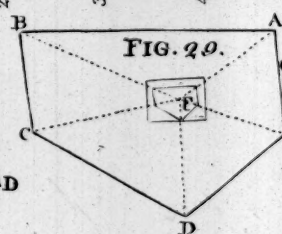


FIG. 29.

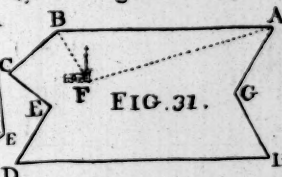


FIG. 30.

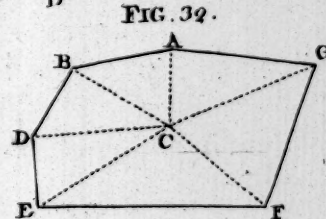


FIG. 31.

FIG. 30.

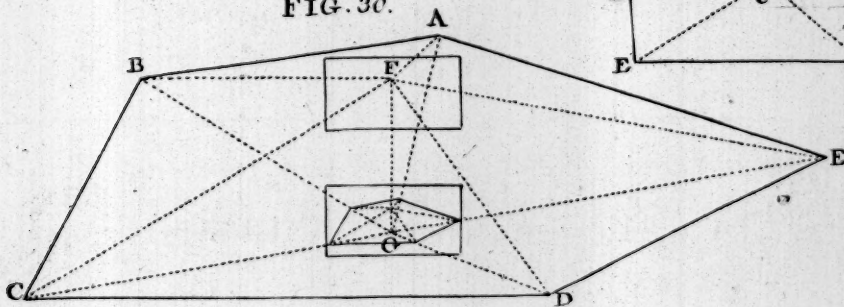


FIG. 32.

After the very same manner, from the sides and angles given, which bound any right-lined figure, a figure like to it may be drawn, and the rest of its sides and angles be known.

C O R O L L A R Y.

Hence any trigonometrical problem in right-lined triangles, may be resolved by delineating the triangle from what is given concerning it, as in this proposition. The unknown sides are examined by a line of equal parts, and the angles by a protractor or line of chords.

P R O P. XXII. PROB.

The diameter of a circle being given, to find its circumference nearly.

THE periphery of any polygon inscribed in the circle is less than the circumference, and the periphery of any polygon

lygon described about a circle is greater than the circumference. When Archimedes first discovered that the diameter was in proportion to the circumference, as 7 to 22 nearly, which serves for common use. But the moderns have computed the proportion of the diameter to the circumference to greater exactness. Supposing the diameter 100, the periphery will be more than 314, but less than 315 *. But Ludolphus Van Cuelen exceeded the labours of all; for, by immense study, he found, that, supposing the diameter

100,000,000,000,000,000,000,000,000,000,

the periphery will be less than

314,159,265,358,979,323,846,264,338,327,951,

but greater than

314,159,265,358,979,323,846,264,338,327,950;

whence it will be easy, any part of the circumference being given in degrees and minutes, to assign it in parts of the diameter.

Of

* The diameter is more nearly to the circumference, as 113 to 355.

Of Surveying and Measuring of Land.

HITHERTO we have treated of the measuring of angles and sides, whence it is abundantly easy to lay down a field, a plane, or an entire country: For to this nothing is requisite but the protraction of triangles, and of other plain figures, after having measured their sides and angles. But, as this is esteemed an important part of Practical Geometry, we shall subjoin here an account of it, with all possible brevity; suggesting withal, that a surveyor will improve himself more by one day's practice, than by a great deal of reading.

P R O P.

PROP. XXIII. PROB.

To Explain what Surveying is, and what Instruments Surveyors use.

FIRST, it is necessary that the surveyor view the field that is to be measured, and investigate its sides and angles, by means of an iron chain, (having a particular mark at each foot of length, or at any number of feet, as may be most convenient for reducing lines or surfaces to the received measures*), and the Graphometer described above. *Secondly*, It is necessary to delineate the field *in plano*, or to form a map of it; that is, to lay down, on paper, a figure similar to the field; which is done by the Protractor, (or line of chords), and of the line of equal parts. *Thirdly*, It is necessary to find out the area of the field so surveyed and represented

* See above, pag. 8. the account of Gunter's chain, and of the chain that is most convenient for measuring land in Scotland.

represented by a map. Of this last we are to treat below in the second part.

The sides and angles of small fields are surveyed by the help of a plain table; which is generally of an oblong rectangular figure, and supported by a *fulcrum*, so as to turn every way, by means of a ball and socket. It is a moveable frame, which surrounds the board, and serves to keep a clean paper put on the board close and tight to it. The sides of the frame facing the paper are divided into equal parts every way. The board hath besides a box with a magnetic needle, and moreover a large index with two sights. On the edge of the frame of the board are marked degrees and minutes, so as to supply the room of a Graphometer.

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P R O P.

P R O P. XXIV.

PROB. FIG. 29.

To delineate a field by the help of a plain table, from one station whence all its angles may be seen, and their distances measured by a chain.

LET the field that is to be laid down be ABCDE. At any convenient place F, let the plain table be erected; cover it with clean paper, in which let some point near the middle represent the station. Then, applying at this place the index with the sights, direct it so as that through the sights some mark may be seen at one of the angles, suppose A; and from the point F, representing the station, draw a faint right line along the side of the index: Then, by the help of the chain, let FA, the distance of the station from the foresaid angle, be measured.

measured. Then, taking what part you think convenient for a foot or pace from the line of equal parts, set off on the faint line the parts corresponding to the line FA that was measured; and let there be a mark made representing the angle of the field A. Keeping the table immoveable, the same is to be done with the rest of the angles; then right lines joining those marks shall include a figure like to the field, as is evident from 5th, 6. Eucl.

C O R O L L A R Y.

The same thing is done in like manner by the Graphometer; for having observed in each of the triangles, AFB, BFC, CFD, &c. the angle at the station F, and having measured the lines from the station to the angles of the field, let similar triangles be protracted on paper, (by the 21. of this), having their common vertex in the point of station. All the lines, excepting those which
represent

represent the sides of the field, are to be drawn faint or obscure.

Note 1. When a surveyor wants to lay down a field, let him place distinctly in a register all the observations of the angles, and the measures of the sides, until, at time and place convenient, he draw out the figure on paper.

Note 2. The observations made by the help of the Graphometer are to be examined; for all the angles about the point F ought to be equal to four right ones, by 13th, 1. Eucl.

PROP.

P R O P. XXV.

P R O B. FIG. 30.

To lay down a field by means of two stations, from each of which all the angles can be seen, by measuring only the distance of the stations.

L E T the instrument be placed at the station F; and having chosen a point representing it upon the paper which is laid upon the plain table, let the index be applied at this point, so as to be moveable about it. Then let it be directed successively to the several angles of the field; and, when any angle is seen through the sights, draw an obscure line along the side of the index. Let the index, with the sights, be directed after the same manner to the station G; on the obscure line, drawn along its side, pointing to A, set off, from the scale of equal parts, a line corresponding to the measured distance

distance of the stations ; and this will determine the point G. Then remove the instrument to the station G ; and applying the index to the line representing the distance of the stations, place the instrument so that the first station may be seen through the sights. Then the instrument remaining immoveable, let the index be applied at the point representing the second station G ; and be successively directed, by means of its sights, to all the angles of the field, drawing (as before) obscure lines ; and the intersection of the two obscure lines that were drawn to the same angle from the two stations will always represent that angle on the plan. Care must be taken that those lines be not mistaken for one another. Lines joining those intersections will form a figure on the paper like to the field.

SCHO-

S C H O L I U M.

It will not be difficult to do the same by the graphometer, if you keep a distinct account of your observations of the angles made by the line joining the stations, and the lines drawn from the stations to the respective angles of the field. And this is the most common manner of laying down whole countries. The tops of two mountains are taken for two stations, and their distance is either measured by some of the methods mentioned above, or is taken according to common repute. The sights are successively directed towards cities, churches, villages, forts, lakes, turnings of rivers, woods, &c.

Note. The distance of the stations ought to be great enough, with respect to the field that is to be measured; such ought to be chosen as are not in a line with any angle of the field. And care ought to be taken likewise, that the angles, for example, FAG , FDG , &c. be neither very acute, nor very obtuse.

obtuse. Such angles are to be avoided as much as possible ; and this admonition is found very useful in practice.

P R O P. XXVI.

P R O B. FIG. 31.

To lay down any field, however irregular its figure may be, by the help of the Graphometer.

LET ABCEDHG be such a field. Let its angles (in going round it) be observed with a graphometer, (by the 12th of this), and noted down; let its sides be measured with a chain; and (by what was said on the 21st of this) let a figure, like to the given field, be protracted on paper. If any mountain is in the circumference, the horizontal line hid under it is to be taken for a side, which may be found by two or three observations, according to some of the methods

thods described above ; and its place on the map is to be distinguished by a shade, that it may be known a mountain is there.

If not only the circumference of the field is to be laid down in the plan, but also its contents, as villages, gardens, churches, public roads, we must proceed in this manner.

Let there be (for example) a church F, to be laid down in the plan. Let the angles ABF, BAF be observed, and protracted on paper in their proper places, the intersection of the two sides BF and AF will give the place of the church on the paper: Or, more exactly, the lines BF, AF being measured, let circles be described from the centres B and A, with parts from the scale corresponding to the distances BF and AF, and the place of the church will be at their intersection.

Note 1. While the angles observed by the graphometer are taken down, you must be careful to distinguish the external angles, as E and G, that they may be rightly protracted afterwards on paper.

I

Note

Note 2. Our observations of the angles may be examined, by computing, if all the internal angles make twice as many right angles, four excepted, as there are sides of the figure : For this is demonstrated by 32d 1. Eucl. But, in place of any external angle DEC, its complement to a circle is to be taken.

PROP. XXVII.

PROB. FIG. 32.

To lay down a plain field without instruments.

IF a small field is to be measured, and a map of it to be made, and you are not provided with instruments ; let it be supposed to be divided into triangles, by right lines, as in the figure ; and after measuring the three sides of any of the triangles, for example, of ABC, let its sides be laid down, from a convenient scale, on paper, by the

22d

22d of this. Again, let the other two sides BD, CD of the triangle CBD be measured, and protracted on the paper, by the same scale as before. In the same manner proceed with the rest of the triangles of which the field is composed ; and the map of the field will be perfected: For the three sides of a triangle determine the triangle; whence each triangle on the paper is similar to its correspondent triangle in the field, and is similarly situated: Consequently, the whole figure is like to the whole field.

S C H O L I U M.

If the field be small, and all its angles may be seen from one station, it may be very well laid down by the plain table, by the 24th of this. If the field be larger, and have the requisite conditions, and great exactness is not expected, it likewise may be plotted by means of the plain table, or by the Graphometer, according to the 25th
of

of this ; but, in fields that are irregular and mountainous, when an exact map is required, we are to make use of the Graphometer, as in the 26th of this, but rarely of the plain table.

Having protracted the bounding lines, the particular parts contained within them may be laid down, by the proper operations for this purpose, delivered in the 26th proposition ; and the method described in the 27th proposition may be sometimes of service ; for we may trust more to the measuring of sides, than to the observing of angles. We are not to compute four-sided and many-sided figures, till they are resolved into triangles : For the sides do not determine those figures.

In the laying down of cities, or the like, we may make use of any of the methods described above that may be most convenient.

The map being finished, it is transferred on clean paper, by putting the first sketch above it, and marking the angles by the point of a small needle. These points being
joined

joined by right lines, and the whole illuminated by colours proper to each part, and the figure of the mariner's compass being added to distinguish the north and south, with a scale on the margin, the map or plan will be finished and neat.

We have thus briefly and plainly treated of surveying, and shown by what instruments it is performed; having avoided those methods which depend on the magnetic needle, not only because its direction may vary in different places of a field, (the contrary of this at least doth not appear), but because the quantity of an angle observed by it cannot be exactly known; for an error of two or three degrees can scarcely be avoided in taking angles by it. As for the remaining part of surveying, whereby the area of a field already laid down on paper, is found in acres, roods, or any other superficial measures; this we leave to the following part, which treats of the mensuration of surfaces.

‘ Besides the instruments described above,
 ‘ a surveyor ought to be provided with an
 ‘ off-

‘ off-set staff, equal in length to ten links of
‘ the chain, and divided into ten equal parts.
‘ He ought likewise to have ten arrows or
‘ small straight sticks, near two feet long,
‘ shod with iron ferrils. When the chain is
‘ first opened, it ought to be examined by
‘ the off-set staff. In measuring any line,
‘ the leader of the chain is to have the ten
‘ arrows at first setting out. When the
‘ chain is stretched in the line, and the near
‘ end touches the place from which you
‘ measure, the leader sticks one of the ten
‘ arrows in the ground, at the far end of
‘ the chain. Then the leader leaving the
‘ arrow, proceeds with the chain another
‘ length ; and the chain being stretched in
‘ the line, so that the near end touches the
‘ first arrow, the leader sticks down another
‘ arrow at his end of the chain. The line
‘ is preserved straight, if the arrows be al-
‘ ways set so as to be in a right line with
‘ the place you measure from, and that to
‘ which you are going. In this manner
‘ they proceed till the leader have no more
‘ arrows.

' arrows. At the eleventh chain, the ar-
 ' rows are to be carried to him again, and
 ' he is to stick one of them in the ground,
 ' at the end of the chain. And the same is
 ' to be done at the 21 31. 41. &c. chains,
 ' if there are so many in the right line to be
 ' measured. In this manner you can hardly
 ' commit an error in numbering the chains,
 ' unless of ten chains at once.

' The off-set staff serves for measuring
 ' readily the distances of any things proper
 ' to be represented in your plan, from the
 ' station-line while you go along. These
 ' distances ought to be entered into your
 ' field-book, with the corresponding distances
 ' from the last station, and proper remarks,
 ' that you may be enabled to plot them
 ' justly, and be in no danger of mistaking
 ' one for another, when you extend your
 ' plan. The field-book may be conveni-
 ' ently divided into five columns. In the
 ' middle column the angles at the several
 ' stations taken by the Theodolite are to be
 ' entered, with the distances from the sta-
 ' tions.

‘ tions. The distances taken by the off-set
 ‘ staff, on either side of the station-line, are
 ‘ to be entered into columns, on either side
 ‘ of the middle column, according to their
 ‘ position with respect to that line. The
 ‘ names or characters of the objects, with
 ‘ proper remarks, may be entered in co-
 ‘ lumns on either side of these last.

‘ Because, in the place of the Grapho-
 ‘ meter described by our author, Surveyors
 ‘ now make use of the Theodolite, we shall
 ‘ subjoin a description of Mr Sisson’s latest
 ‘ improved Theodolite from Mr Gardiner’s
 ‘ Practical Surveying improved. See a fi-
 ‘ gure of it in plate 4.

‘ In this instrument, the three staffs, by
 ‘ brass ferrils at top, screw into bell-metal
 ‘ joints, that are moveable between brass
 ‘ pillars, fixed on a strong brass plate ; in
 ‘ which, round the centre, is fixed a foc-
 ‘ ket with a ball moveable in it, and upon
 ‘ which the four screws press, that set the
 ‘ limb horizontal : Next above is another
 ‘ such plate, through which the said screws
 ‘ pass, and on which, round the centre, is
 ‘ fixed

' fixed a frustum of a cone of bell-metal,
 ' whose axis (being connected with the
 ' centre of the ball) is always perpendicular
 ' to the limb, by means of a conical brass
 ' ferril fitted to it, whereon is fixed the
 ' compass-box ; and on it the limb, which
 ' is a strong bell-metal ring, whereon are
 ' moveable three brass indexes ; in whose
 ' plate are fixed four brass pillars, that,
 ' joining at top, hold the centre-pin of the
 ' bell-metal double sextant, whose double
 ' index is fixed on the centre of the same
 ' plate : Within the double sextant is fixed
 ' the spirit-level, and over it the telescope.

' The compass-box is graved with two
 ' diamonds for north and south, and with
 ' 20 degrees on both sides of each, that the
 ' needle may be set to the variation, and its
 ' error also known.

' The limb is two *Fleurs de luce* against
 ' the diamonds in the box, instead of 180
 ' each ; and is curiously divided into whole
 ' degrees, and numbered to the left hand
 ' at every ten to twice 180, having three

K

' indexes

‘ indexes distant 120, (with Nonius’s divi-
‘ fions on each for the decimals of a de-
‘ gree), that are moved by a pinion fixed
‘ below one of them, without moving the
‘ limb; and in another is a screw and
‘ spring under, to fix it to any part of the
‘ limb. It has also divisions numbered,
‘ for taking the quarter girt in inches of
‘ round timber at the middle height, when
‘ standing ten feet horizontally distant from
‘ its centre; which at 20 must be doubled,
‘ and at 30 tripled; to which a shorter in-
‘ dex is used, having Nonius’s divisions for
‘ the decimals of an inch; but an abatement
‘ must be made for the bark, if not taken off.

‘ The double sextant is divided on one
‘ side from under its centre (when the spi-
‘ rit-tube and telescope are level) to above
‘ 60 degrees each way, and numbered at
‘ 10. 20. &c. and the double index (through
‘ which it is moveable) shews on the same
‘ side the degree and decimal of any alti-
‘ tude or depression to that extent by No-
‘ nius’s divisions: On the other side are di-
‘ visions

' visions numbered for taking the upright
 ' height of timber, &c. in feet, when distant
 ' 10 feet ; which at 20 must be doubled,
 ' and at 30 tripled ; and also the quantities
 ' for reducing hypotenusal lines to hori-
 ' zontal. It is moveable by a pinion fixed
 ' in the double index.

' The telescope is a little shorter than
 ' the diameter of the limb, that a fall may
 ' not hurt it ; yet it will magnify as much,
 ' and shew a distant object as perfect, as
 ' most of triple its length. In its focus are
 ' very fine cross wires, whose intersection
 ' is in the plane of the double sextant ; and
 ' this was a whole circle, and turned in a
 ' lathe to a true plane, and is fixed at right
 ' angles to the limb ; so that, whenever the
 ' limb is set horizontal, (which is readily
 ' done by making the spirit-tube level over
 ' two screws, and the like over the other
 ' two), the double sextant and telescope are
 ' moveable in a vertical plane ; and then
 ' every angle taken on the limb (though
 ' the telescope be never so much elevated
 ' or

‘ or depressed) will be an angle in the plane
 ‘ of the horizon. And this is absolutely
 ‘ necessary in plotting a horizontal plane.

‘ If the lands to be plotted are hilly, and
 ‘ not in any one plane, the lines measured
 ‘ cannot be truly laid down on paper, with-
 ‘ out being reduced to one plane, which
 ‘ must be the horizontal, because angles are
 ‘ taken in that plane.—

‘ In viewing my objects, if they have
 ‘ much altitude or depression, I either write
 ‘ down the degree and decimal shewn on
 ‘ the double sextant, or the links shewn on
 ‘ the back-side; which last subtracted from
 ‘ every chain in the station-line, leaves the
 ‘ length in the horizontal plane. But, if
 ‘ the degree is taken, the following table
 ‘ will shew the quantity.

A TABLE

A TABLE of the links to be subtracted out of every chain in hypthenusal lines of several degrees altitude, or depression, for reducing them to horizontal.

Degrees. Links.	Degrees. Links.	Degrees. Links.
4,05 — $\frac{1}{4}$	14,07 — 3	23,074 — 8
5,73 — $\frac{1}{2}$	16,26 — 4	24,495 — 9
7,02 — $\frac{3}{4}$	18,195 — 5	25,84 — 10
8,11 — 1	19,95 — 6	27,13 — 11
11,48 — 2	21,565 — 7	28,36 — 12

‘ Let the first station-line really measure
 ‘ 1107 links, and the angle of altitude or
 ‘ depression be $19^{\circ}, 95$; looking in the table
 ‘ I find against $19^{\circ}, 95$, is six links. Now
 ‘ 6 times 11 is 66; which subtracted from
 ‘ 1107, leaves 1041, the true length to be
 ‘ laid down in the plan.

‘ It is useful in surveying, to take the
 ‘ angles, which the bounding lines form,
 ‘ with the magnetic needle, in order to check
 ‘ the angles of the figure, and to plot them
 ‘ conveniently afterwards.’

PART

PART II.

Of the Surfaces of Bodies.

THE smallest superficial measure with us is a square inch ; 144 of which makes a square foot. Wrights make use of these in the measuring of deals and planks ; but the square foot which the glaziers use in measuring of glafs, consists only of 64 square inches. The other measures are, *first*, the ell square ; *2dly*, the fall, containing 36 square ells ; *3dly*, the rood, containing 40 falls ; *4thly*, the acre, containing 4 roods. Slaters, masons, and pavers, use the ell square and the fall ; surveyors of land use the square ell, the fall, the rood, and the acre.

The superficial measures of the English, are *first*, the square foot ; *2dly*, the square yard, containing 9 square feet ; for their yard contains only 3 feet ; *3dly*, the pole containing $30\frac{1}{4}$ square yards ; *4thly*, the rood,

rood, containing 40 poles ; *5thly*, the acre, containing 4 roods. And hence it is easy to reduce our superficial measures to the English, or theirs to our.

‘ In order to find the content of a field,
 ‘ it is most convenient to measure the lines
 ‘ by the chains described above, p. 4. that
 ‘ of 22 yards for computing the English
 ‘ acres, and that of 24 Scots ells for the
 ‘ acres of Scotland. The chain is divided
 ‘ into 100 links, and the square of the chain
 ‘ is 10,000 square links ; ten squares of the
 ‘ chain, or 100,000 square links give an
 ‘ acre. Therefore, if the area be expressed
 ‘ by square links, divided by 100,000, or
 ‘ cut off five decimal places, and the quo-
 ‘ tient shall give the area in acres and de-
 ‘ cimals of an acre. Write the entire acres
 ‘ apart ; but multiply the decimals of an
 ‘ acre by 4, and the product shall give the
 ‘ remainder of the area in roods and deci-
 ‘ mals of a rood. Let the entire roods be
 ‘ noted apart after the acres ; then multiply
 ‘ the decimals of a rood by 40, and the
 ‘ product

' product shall give the remainder of the
 ' area in falls or poles. Let the entire falls
 ' or poles be then writ after the roods, and
 ' multiply the decimals of a fall by 36, if
 ' the area is required in the measures of
 ' Scotland ; but multiply the decimals of a
 ' pole by $30\frac{1}{4}$, if the area is required in the
 ' measures of England, and the product
 ' shall give the remainder of the area in
 ' square ells in the former case, but in square
 ' yards in the latter. If, in the former case,
 ' you would reduce the decimals of the
 ' square ell to square feet, multiply them by
 ' 9.50694 ; but, in the latter case, the de-
 ' cimals of the English square yard are re-
 ' duced to square feet, by multiplying them
 ' by 9.

' Suppose, for example, that the area ap-
 ' pears to contain 12.65842 square links of
 ' the chain of 24 ells ; and that this area is
 ' to be expressed in acres, roods, falls, &c.
 ' of the measures of Scotland. Divide the
 ' square links by 100,000, and the quotient
 ' 12.65842 shows the area to contain 12
 ' acres

' acres $\frac{65842}{100000}$ of an acre. Multiply the de-
 ' cimal part by 4, and the product 2.63368
 ' gives the remainder in roods and decimals
 ' of a rood. Those decimals of the rood
 ' being multiplied by 40, the product gives
 ' 25.3472 falls. Multiply the decimals of
 ' the fall by 36, and the product gives
 ' 12.4992 square ells. The decimals of
 ' the square ell multiplied by 9.50994 give
 ' 4.7458 square feet. Therefore the area
 ' proposed amounts to 12 acres, 2 roods, 25
 ' falls, 12 square ells, and 4 $\frac{7458}{100000}$ square
 ' feet.

' But, if the area contains the same
 ' number of square links of Gunter's chain,
 ' and is to be expressed by English mea-
 ' sures, the acres and roods are computed
 ' in the same manner as in the former case.
 ' The poles are computed as the falls. But
 ' the decimals of the pole, viz. $\frac{3472}{100000}$, are
 ' to be multiplied by $30\frac{1}{2}$ (or 30.25), and
 ' the product gives 10.5028 square yards.
 ' The decimals of the square yard, multi-
 ' plied by 9, give 4.5252 square feet;

L

' there-

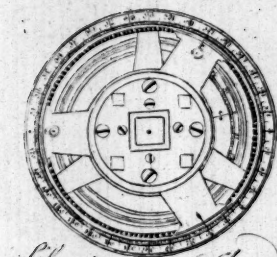
‘ therefore, in this case, the area is in English measure 12 acres, 3 roods, 25 poles, 10 square yards, and $4 \frac{5}{1000000}$ square feet.

‘ The Scots acre is to the English acre, by statute, as 100,000 to 78,694, if we have regard to the difference betwixt the Scots and English foot above mentioned. But it is customary in some parts of England to have 18,21, &c. feet to a pole, and 160 such poles to an acre; whereas, by the statute, $16\frac{1}{2}$ feet make a pole. In such cases the acre is greater in the duplicate ratio of the number of feet to a pole.

‘ They who measure land in Scotland by an ell of 37 English inches, make the acre less than the true Scots acre by $593\frac{8}{100}$ square English feet, or by about $\frac{1}{93}$ of the acre.

‘ An husband-land contains 6 acres of sock and scythe-land, that is, of land that may be tilled with a plough, and mown with a scythe; 13 acres of arable land make an oxgang or oxengate; four oxengate make a pound-land of old extent (by

‘ a



Siboni's Shoodolite.

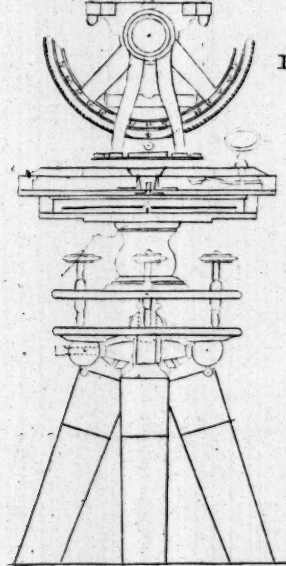


FIG. 8.

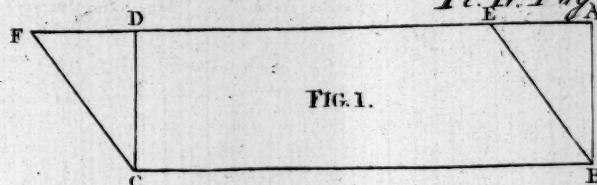
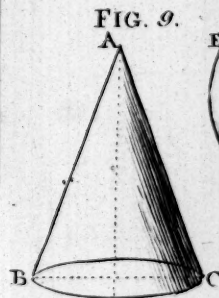
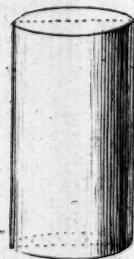


FIG. 1.

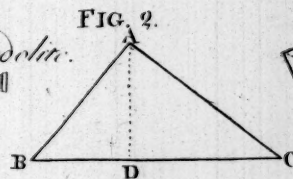


FIG. 2.

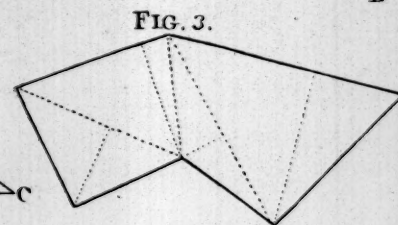


FIG. 3.

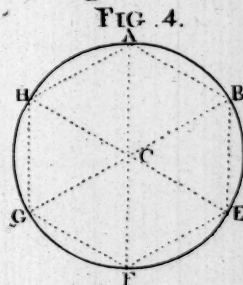


FIG. 4.

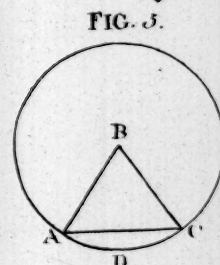


FIG. 5.

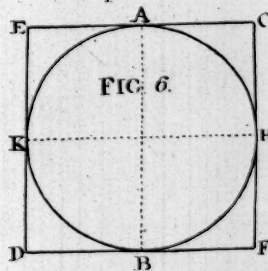


FIG. 6.

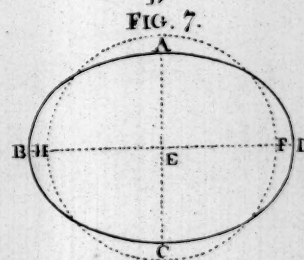


FIG. 7.

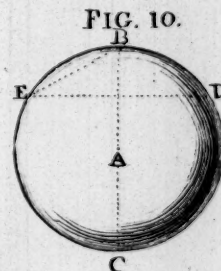


FIG. 10.

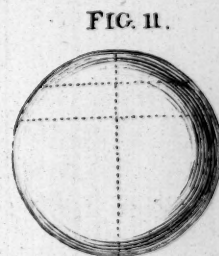


FIG. 11.

‘ a decree of the exchequer, March 11.
 ‘ 1585), and is called *librato terrae*. A for-
 ‘ ty shilling land of old extent contains
 ‘ eight oxgang, or 104 acres.

‘ The Arpent about Paris contains 32400
 ‘ square Paris feet, and is equal to $2\frac{2}{3}$ Scots
 ‘ roods, or $3\frac{3}{10}$ English roods.

‘ The *Ætus quadratus*, according to Var-
 ‘ ro, Collumella, &c. was a square of 120
 ‘ Roman feet: The jugerum was the double
 ‘ of this. ’Tis to the Scots acre as 10,000
 ‘ to 20,456, and to the English acre as
 ‘ 10,000 to 16,097. It was divided (like
 ‘ the As) into 12 unciae, and the uncia in-
 ‘ to 24 scrupula.’ This, with the three
 preceding paragraphs, are taken from an
 ingenious manuscript written by Sir Robert
 Stewart professor of natural philosophy.
 The greatest part of the table in p. 6. was
 taken from it likewise.

PROP.

PROP. I.

PROB. FIG. 1.

To find out the area of a rectangular parallelogram ABCD.

LET the side AB, for example, be 5 feet long, and BC (which constitutes with AB a right angle at B) be 17 feet. Let 17 be multiplied by 5, and the product 85 will be the number of square feet in the area of the figure ABCD. But, if the parallelogram proposed is not rectangular, as BEFC, its base BC multiplied into its perpendicular height AB (not into its side BE) will give its area. This is evident from 35th 1. Eucl.

PROP.

P R O P. II.

P R O B. FIG. 2.

To find the Area of a given Triangle.

LET the triangle BAC be given, whose base BC is supposed 9 feet long: Let the perpendicular AD be drawn from the angle A opposite to the base, and let us suppose AD to be four feet. Let the half of the perpendicular be multiplied into the base, or the half of the base into the perpendicular, or take the half of the product of the whole base into the perpendicular, the product gives 18 square feet for the area of the given triangle.

But, if only the sides are given, the perpendicular is found either by protracting the triangle, or by 12th and 13th 2. Eucl. or by trigonometry. But how the area of a triangle may be found from the given sides only, shall be shewn in the 4th prop. of this part.

P R O P.

P R O P. III.

P R O B. FIG. 3.

To find the Area of any rectilineal Figure.

IF the figure be irregular, let it be resolved into triangles ; and drawing perpendiculars to the bases in each of them, let the area of each triangle be found by the preceding Prop. and the sum of these areas will give the area of the figure.

S C H O L I U M I.

In measuring boards, planks, and glass, their sides are to be measured by a foot-rule divided into 100 equal parts ; and after multiplying the sides, the decimal fractions are easily reduced to lesser denominations. The mensuration of these is easy, when they are rectangular parallelograms.

S C H O-

SCHOLIUM 2.

If a field is to be measured, let it first be plotted on paper, by some of the methods described in the preceding part, and let the figure so laid down be divided into triangles, as was shown in the preceding proposition.

The base of any triangle, or the perpendicular upon the base, or the distance of any two points of the field, is measured by applying it to the scale according to which the map is drawn.

SCHOLIUM 3.

But, if the field given be not in a horizontal plane, but uneven and mountainous, the scale gives the horizontal line between any two points, but not their distance measured on the uneven surface of the field. And indeed it would appear that the horizontal plane is to be accounted the area of an uneven and hilly country. For, if such ground

ground is laid out for building on, or for planting with trees, or bearing corn, since these stand perpendicular to the horizon, it is plain that a mountainous country cannot be considered as of greater extent for those uses than the horizontal plane ; nay, perhaps, for nourishing of plants, the horizontal plane may be preferable.

If, however, the area of a figure, as it lies irregularly on the surface of the earth, is to be measured, this may be easily done by resolving it into triangles as it lies. The sum of their areas will be the area sought ; which exceeds the area of the horizontal figure more or less, according as the field is more or less uneven.

PROP.

P R O P. IV.

P R O B. FIG. 2.

The sides of a triangle being given, to find the area, without finding the perpendicular.

LET all the sides of the triangle be collected into one sum; from the half of which let the sides be separately subtracted, that three differences may be found betwixt the foresaid half sum and each side; then let these three differences and the half sum be multiplied into one another, and the square root of the product will give the area of the triangle. For example, let the sides be 10, 17, 21; the half of their sum is 24; the three differences betwixt this half sum and the three sides, are 14, 7, and 3. The first being multiplied by the second, and their product by the third, we have 294 for the product of the differences; which

M

mul-

multiplied by the foresaid half sum 24, gives 7056; the square root of which 84 is the area of the triangle. The demonstration of this, for the sake of brevity, we omit. It is to be found in several treatises, particularly in Clavius's Practical Geometry.

P R O P. V.

T H E O R. FIG. 4.

The area of the ordinate figure ABEFGH is equal to the product of the half circumference of the polygon, multiplied into the perpendicular drawn from the centre of the circumscribed circle to the side of the polygon.

FOR the ordinate figure can be resolved into as many equal triangles, as there are sides of the figure; and, since each triangle is equal to the product of half the base into the perpendicular, it is evident that
the

the sum of all the triangles together, that is, the polygon, is equal to the product of half the sum of the bases (that is, the half of the circumference of the polygon) into the common perpendicular height of the triangles drawn from the centre C to one of the sides ; for example, to AB.

P R O P. VI.

P R O B. FIG. 5.

The area of a circle is found by multiplying the half of the periphery into the radius, or the half of the radius into the periphery.

FOR a circle is not different from an ordinate or regular polygon of an infinite number of sides, and the common height of the triangles into which the polygon or circle may be supposed to be divided is the radius of the circle.

Were

Were it worth while, it were easy to demonstrate accurately this proposition, by means of the inscribed and circumscribed figures, as is done in the 5th Prop. of the treatise of Archimedes concerning the dimensions of the circle.

C O R O L L A R Y.

Hence also it appears that the area of the sector ABCD is produced, by multiplying the half of the arch into the radius ; and likewise that the area of the segment of the circle ADC is found, by subtracting from the area of the sector the area of the triangle ABC.

P R O P. VII.

T H E O R. FIG. 6.

The circle is to the square of the diameter, as 11 to 14 nearly.

FOR, if the diameter AB be supposed to be 7, the circumference AHBK will be almost 22 (by the 22d Prop. of the first part

part of this), and the area of the square DC will be 49 ; and, by the preceding Prop. of this, the area of the circle will be $38\frac{1}{2}$: Therefore the square DC will be to the inscribed circle as 49 to $38\frac{1}{2}$, or as 98 to 77, that is, as 14 to 11. Q. E. D.

If greater exactness is required, you may proceed to any degree of accuracy : For the square DC is to the inscribed circle, as 1 to $1 - \frac{1}{3} + \frac{1}{5} - \frac{1}{7} + \frac{1}{9} - \frac{1}{11} + \frac{1}{13}$, &c. *in infinitum*.

‘ This series will be of no service for
 ‘ computing the area of the circle accurately,
 ‘ without some further artifice, because it
 ‘ converges at too slow a rate. The area
 ‘ of the circle will be found exactly enough
 ‘ for most purposes, by multiplying the
 ‘ square of the diameter by 7854, and divi-
 ‘ ding by 10,000, or cutting off four deci-
 ‘ mal places from the product ; for the area
 ‘ of the circle is to the circumscribed square
 ‘ nearly as 7854 to 10,000.’

PROP.

P R O P. VIII.

PROB. FIG. 7.

To find the Area of a given Ellipse.

LET ABCD be an ellipse, whose greater diameter is BD, and lesser AC, bisecting the greater perpendicularly in E. Let a mean proportional HF be found (by 13th 6. Eucl.) between AC and BD, and (by the 6th of this) find the area of the circle described on the diameter HF. I say, that this area is equal to the area of the ellipse ABCD. For because, as BD to AC, so the square of BD to the square of HF, (by 2. Cor. 20th 6. Eucl.): But, (by the 2d 12. Eucl.) as the square of BD to the square of HF, so is the circle of the diameter BD to the circle of the diameter HF: Therefore, as BD to AC, so is the circle of the diameter BD to the circle of the diameter HF. And (by the 5th Prop. of Archimedes of spheroids) as the
the

the greater diameter BD to the lesser AC, so is the circle of the diameter BD to the ellipse ABCD. Consequently (by the 11th 5. Eucl.) the circle of the diameter BD will have the same proportion to the circle of the diameter HF, and to the ellipse ABCD. Therefore, by 9th 5. Eucl. the area of the circle of the diameter HF will be equal to the area of the ellipse ABCD. *Q. E. D.*

S C H O L I U M.

From this and the two preceding propositions, a method is derived of finding the area of an ellipse. There are two ways :
 1st, Say, as one is to the lesser diameter, so is the greater diameter to a fourth number, (which is found by the rule of three.) Then again say, as 14 to 11, so is the 4th number found to the area sought. But the second way is shorter. Multiply the lesser diameter into the greater, and the product by 11 ; then divide the whole product by 14, and the quotient will be the area sought
 of

of the ellipse. For example, Let the greater diameter be 10, and the lesser 7, by multiplying 10 by 7, the product is 70; and multiplying that by 11, it is 770; and dividing 770 by 14, the quotient will be 55, which is the area of the ellipse sought.

‘The area of the ellipse will be found more accurately, by multiplying the product of the two diameters by 7854.’

We shall add no more about other plain surfaces, whether rectilinear or curvilinear, which seldom occur in practice; but shall subjoin some propositions about measuring the surfaces of solids.

PROP.

PROP. IX. PROB.

To measure the surface of any Prism.

BY the 14th definition of the 11th Eucl. a prism is contained by planes, of which two opposite sides (commonly called the bases) are plain rectilineal figures; which are either regular and ordinate, and measured by Prop. 5. of this part; or however irregular, and then they are measured by the 3d Prop. of this book. The other sides are parallelograms, which are measured by the 1st Prop. of the second part; and the whole superficies of the prism consists of the sum of those taken altogether.

PROP. X. PROE.

To measure the superficies of any Pyramid.

SINCE its basis is a rectilineal figure, and the rest of the planes terminating in the top of the pyramid are triangles; N these

these measured separately, and added together, give the surface of the pyramid required.

PROP. XI. PROB.

To measure the superficies of any Regular Body.

THOSE bodies are called regular, which are bounded by equilateral and equiangular figures. The superficies of the tetraedron consists of four equal and equiangular triangles; the superficies of the hexaedron, or cube, of six equal squares; an octaedron, of eight equal equilateral triangles; a dodecaedron, of twelve equal and ordinate pentagons; and the superficies of an icosaedron, of twenty equal and equilateral triangles. Therefore it will be easy to measure these surfaces from what has been already shown.

In the same manner we may measure the superficies of a solid contained by any planes.

PROP.

P R O P. XII.

P R O B. FIG. 8.

To measure the Superficies of a Cylinder.

BECAUSE a cylinder differs very little from a prism, whose opposite planes (or bafes) are ordinate figures of an infinite number of fides, it appears that the superficies of a cylinder, without the bafes, is equal to an infinite number of parallelograms; the common altitude of all which is the height of the cylinder, and the basis of them all differ very little from the periphery of the circle, which is the base of the cylinder. Therefore this periphery multiplied into the common height, gives the superficies of the cylinder, excluding the bafes; which are to be measured separately by the help of the 6th Prop. of this part.

This proposition concerning the measure of the surface of the cylinder (excluding

its

its basis) is evident from this, That, when it is conceived to be spread out, it becomes a parallelogram, whose base is the periphery of the circle of the base of the cylinder stretched into a right line, and whose height is the same with the height of the cylinder.

P R O P. XIII.

P R O B. FIG. 9.

To measure the Surface of a Right Cone,

THE surface of a right cone is very little different from the surface of a right pyramid, having an ordinate polygon for its base, of an infinite number of sides; the surface of which (excluding the base) is equal to the sum of the triangles. The sum of the bases of these triangles is equal to the periphery of the circle of the base, and the common height of the triangles is the side of the cone AB: Wherefore the sum of these

these triangles is equal to the product of the sum of the bases (*i. e.* the periphery of the base of the cone) multiplied into the half of the common height, or it is equal to the product of the periphery of the base.

If the area of the bases is likewise wanted, it is to be found separately by the 6th Prop. of this part. If the surface of a cone is supposed to be spread out on a plane, it will become a sector of a circle, whose radius is the side of the cone; and the arch terminating the sector is made from the periphery of the base. Whence, by Corol. 6. Prop. of this, its dimension may be found.

C O R O L L A R Y.

Hence it will be easy to measure the surface of a *frustum* of a cone cut by a plane parallel to the base. As to what relates to the measuring of the surface of the scalenous cone, because it is not very useful in practice, we shall not describe the method;
which

which would carry us beyond the limits of this treatise.

P R O P. XIV.

PROB. FIG. 10.

To measure the Surface of a given Sphere.

LET there be a sphere, whose centre is A, and let the area of its convex surface be required. Archimedes demonstrates (37. Prop. 1. book of the sphere and cylinder) that its surface is equal to the area of four great circles of the sphere; that is, let the area of the great circle be multiplied by 4, and the product will give the area of the sphere; or, by the 20th 6. and 2d 12. of Eucl. the area of the sphere given is equal to the area of a circle whose radius is the right line BC, the diameter of the sphere. Therefore, having measured (by 6th Prop. of this part) the circle described with the radius BC, this will give the surface of the sphere.

P R O P.

P R O P. XV.

P R O B. FIG. 10.

To measure the surface of a segment of a Sphere.

LET there be a segment cut off by the plane ED. Archimedes demonstrates (49. and 50. 1. *de sphaera*) that the surface of this segment, excluding the circular base, is equal to the area of a circle whose radius is the right line BE drawn from the vertex B of the segment to the periphery of the circle DE. Therefore, by the 6th Prop. of this part, it is easily measured.

C O R O L L A R Y I.

Hence that part of the surface of a sphere that lieth between two parallel planes is easily measured, by subtracting the surface of the lesser segment from the surface of the greater segment.

C O R O L-

C O R O L L A R Y 2.

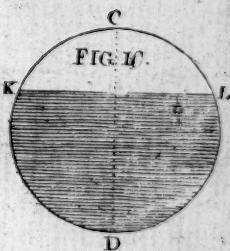
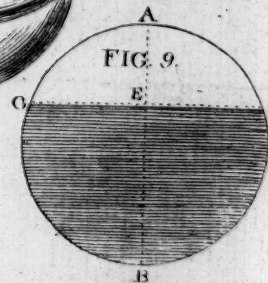
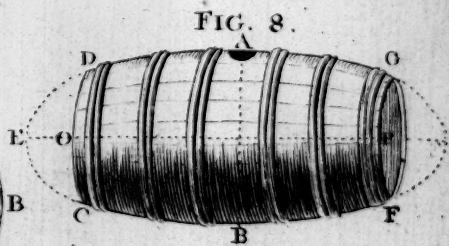
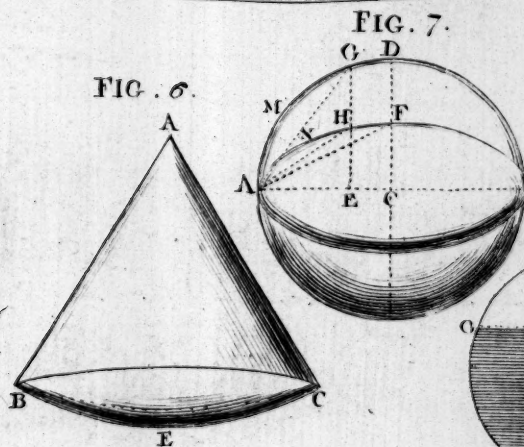
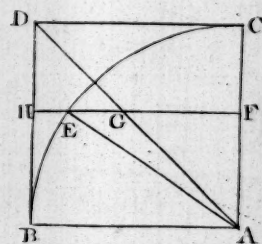
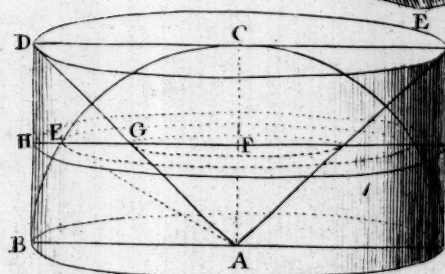
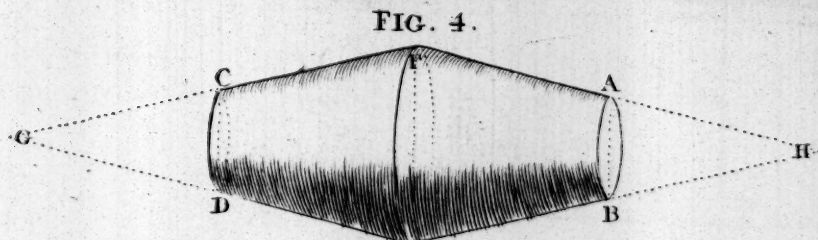
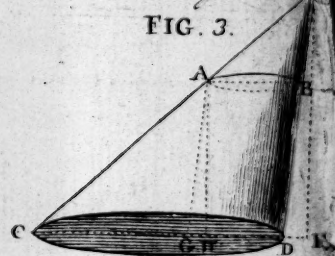
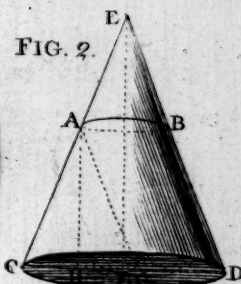
Hence likewise it follows, that the surface of a cylinder, described about a sphere (excluding the basis) is equal to the surface of the sphere, and the parts of the one to the parts of the other, intercepted between planes parallel to the basis of the cylinder.

P A R T III.

Of Solid Figures, and their Mensuration.

AS in the preceding parts we took an inch for the smallest measure in length, and an inch square for the smallest superficial measure; so now, in treating of the mensuration of solids, we take a cubical inch for the smallest solid measure. Of these 109 makes a Scots pint; other liquid measure depends on this, as is generally known.

In dry measures, the firloft, by statute, contains $19\frac{1}{2}$ pints; and on this depend the
other



other dry measures: Therefore, if the content of any solid be given in cubical inches, it will be easy to reduce the same to the common liquid or dry measures, and, conversely, to reduce these to solid inches. The liquid and dry measures in use among other nations, are known from their writers.

‘ As to the English liquid measures, by
 ‘ act of parliament 1706, any round vessel,
 ‘ commonly called a cylinder, having an
 ‘ even bottom, being seven inches in diameter
 ‘ throughout, and six inches deep from
 ‘ the top of the inside to the bottom (which
 ‘ vessel will be found, by computation, to
 ‘ contain $230 \frac{207}{1000}$ cubical inches); or any
 ‘ vessel containing 231 cubical inches, and
 ‘ no more, is deemed to be a lawful wine-
 ‘ gallon. An English pint therefore con-
 ‘ tains $28 \frac{7}{8}$ cubical inches; two pints makes
 ‘ a quart; four quarts a gallon; 18 gallons
 ‘ a roundlet; three roundlets and an half,
 ‘ or 63 gallons, make a hoghead; the half
 ‘ of a hoghead is a barrel; one hoghead

O

‘ and

‘ and a third, or 84 gallons, make a pun-
 ‘ cheon; one puncheon and a half, or two
 ‘ hogheads, or 126 gallons, make a pipe
 ‘ or butt; the third part of a pipe, or 42
 ‘ gallons, make a tierce; two pipes, or three
 ‘ puncheons, or four hogheads, make a ton
 ‘ of wine. Though the English wine gal-
 ‘ lon is now fixed at 231 cubical inches,
 ‘ the standard kept at Guildhall being mea-
 ‘ sured, before many persons of distinction,
 ‘ May 25. 1688, it was found to contain
 ‘ only 224 such inches.

‘ In the English beer measure, a gallon
 ‘ contains 282 cubical inches; consequently
 ‘ $35\frac{1}{4}$ cubical inches make a pint, two pints
 ‘ make a quart, four quarts make a gallon,
 ‘ nine gallons a firkin, four firkins a barrel.
 ‘ In ale, eight gallons make a firkin, and 32
 ‘ gallons make a barrel. By an act of the
 ‘ first of William and Mary, 34 gallons is
 ‘ the barrel, both for beer and ale, in all
 ‘ places, except within the weekly bills of
 ‘ mortality.

‘ In

' In Scotland, it is known that four gills
 ' make a mutchkin, two mutchkins make
 ' chopin, a pint is two chopins, a quart is
 ' two pints, and a gallon is four quarts or
 ' eight pints. The accounts of the cubical
 ' inches contained in the Scots pint vary
 ' considerably from each other. According
 ' to our author, it contains 109 cubical
 ' inches. But the standard-jugs kept by
 ' the Dean-of-guild of Edinburgh (one of
 ' which has the year 1555, with the arms
 ' of Scotland, and the town of Edinburgh,
 ' marked upon it) having been carefully
 ' measured several times, and by different
 ' persons, the Scots pint, according to those
 ' standards, was found to contain about
 ' $103\frac{4}{5}$ cubic inches. The pewterers jugs
 ' (by which the vessels in common use are
 ' made) are said to contain sometimes be-
 ' twixt 105 and 106 cubic inches. A cask
 ' that was measured by the brewers of E-
 ' dinburgh, before the Commissioners of
 ' Excise in 1707, was found to contain $46\frac{7}{8}$
 ' Scots pints; the same vessel contained $18\frac{1}{2}$
 ' English

' English ale-gallons. Supposing this men-
 ' surating to be just, the Scots pint will be
 ' to the English ale-gallon as 289 to 750;
 ' and, if the English ale-gallon be supposed
 ' to contain 282 cubical inches, the Scots
 ' pint will contain 108.664 cubical inches.
 ' But it is suspected, on several grounds,
 ' that this experiment was not made with
 ' sufficient care and exactness.

' The Commissioners appointed by autho-
 ' rity of Parliament to settle the measures
 ' and weights, in their act of February 19.
 ' 1618, relate, That, having caused fill the
 ' Linlithgow firloft with water, they found
 ' that it contained $21\frac{1}{4}$ pints of the just
 ' Stirling jug and measure. They likewise
 ' ordain, that this shall be the just and only
 ' firloft, and add, *That the widenefs and*
 ' *breadth of the which firloft, under and*
 ' *above, even over within the buirds, shall*
 ' *contain nineteen inches and the sixth part*
 ' *of an inch, and the deepness seven inches and*
 ' *a third part of an inch.* According to this
 ' act (supposing their experiment and com-
 ' putation

'putation to have been accurate) the pint
 'contained only 99.56 cubical inches; for
 'the content of such a vessel as is described
 'in the act, is 2115.85, and this divided by
 '21 $\frac{1}{4}$, gives 99.56. But, by the weight of
 'water said to fill this firloft in the same act,
 'the measure of the pint agrees nearly with
 'the Edinburgh standard above mentioned.

'As for the English measures of corn,
 'the Winchester gallon contains 272 $\frac{1}{4}$ cu-
 'bical inches, two gallons make a peck, four
 'pecks, or eight gallons, (that is, 2178 cu-
 'bical inches) make a bushel, and a quarter
 'is eight bushels.

'Our author says, that 19 $\frac{1}{2}$ Scots pints
 'make a firloft. But this does not appear to
 'be agreeable to the statute above mention-
 'ed, nor to the standard-jugs. It may be
 'conjectured that the proportion assigned
 'by him has been deduced from some ex-
 'periment of how many pints, according
 'to common use, were contained in the
 'firloft. For, if we suppose those pints to
 'have been each of 108.664 cubical inches,
 'according

' according to the experiment made in the
 ' 1707 before the Commissioners of Excise,
 ' described above; then $19\frac{1}{2}$ such pints will
 ' amount to 2118.94 cubical inches, which
 ' agrees nearly with 2115.85, the measure
 ' of the firloft by ftatute above mentioned.
 ' But it is probable, that in this he follow-
 ' ed the act 1587, where it is ordained,
 ' That the wheat-firloft fhall contain 19
 ' pints and two joucattes. A wheat-firloft,
 ' marked with the Linlithgow ftamps, be-
 ' ing meafured, was found to contain about
 ' 2211 cubical inches. By the ftatute of
 ' 1618 the barley-firloft was to contain 31
 ' pints of the juft Stirling jug.

' A Paris pint is 48 cubical Paris inches,
 ' and is nearly equal to an Englifh wine
 ' quart. The *Boiffeau* contains 644.68099
 ' Paris cubical inches, or 780.36 Englifh
 ' cubical inches.

' The Roman *Amphora* was a cubical
 ' Roman foot, the *Congius* was the eighth
 ' part of the *Amphora*, the *Sextarius* was
 ' one fixth of the *Congius*. They divided
 ' the

PRACTICAL GEOMETRY. III

‘ the *Sextarius* like the *As* or *Libra*. Of
 ‘ dry measures the *Medimnus* was equal to
 ‘ two *Amphoras*, that is, about $1\frac{2}{5}$ English
 ‘ legal bushels; and the *Modius* was the
 ‘ third part of the *Amphora*.’

P R O P. I. PROB.

To find the solid content of a given Prism.

BY the 2d Prop. of the 2d Part of this,
 let the area of the base of the prism
 be measured, and be multiplied by the
 height of the prism, the product will give
 the solid content of the prism.

P R O P. II. PROB.

To find the solid content of a given Pyramid.

THE area of the base being found, (by
 the 3d Prop. of the 2d Part), let it be
 multiplied by the third part of the height
 of

of the pyramid, or the third part of the base by the height, the product will give the solid content, by 7th, 12. Eucl.

C O R O L L A R Y.

If the solid content of a *frustum* of a pyramid is required, first let the solid content of the entire pyramid be found; from which subtract the solid content of the part that is wanting, and the solid content of the broken pyramid will remain.

P R O P. III. PROB.

To find the content of a given Cylinder.

THE area of the base being found, (by Prop. 6. of the second part), if it be a circle, and by Prop. 8. if it be an ellipse, (for in both cases it is a cylinder, multiply it

it by the height of the cylinder, and the solid content of the cylinder will be produced.

C O R O L L A R Y FIG. 1.

And in this manner may be measured the solid contents of vessels and casks not much different from a cylinder, as ABCD. If towards the middle EF it be somewhat grosser, the area of the circle of the base being found (by 6th Prop. of the 2d Part) and added to the area of the middle circle EF, and the half of their sum (that is, an arithmetical mean between the area of the base, and the area of the middle circle) taken for the base of the vessel, and multiplied into its height, the solid content of the given vessel will be produced.

Note, That the length of the vessel, as well as the diameter of the base, and of the circle EF, ought to be taken within the

P

staves;

114 A TREATISE OF

flaves; for it is the content within the flaves that is fought.

P R O P. IV. PROB.

To find the solid content of a given Cone.

LET the area of the base (found by Prop. 6. 2d Part) be multiplied into $\frac{2}{3}$ of the height, the product will give the solid content of the cone; for, by 10th 12. Eucl. a cone is the third part of a cylinder that has the same base and height.

P R O P.

P R O P. V.

P R O B. FIG. 2. and 3.

To find the solid content of a frustum of a cone cut by a plane parallel to the plane of the base.

FIRST, let the height of the entire cone be found, and thence (by the preceding Prop.) its solid content; from which subtract the solid content of the cone cut off at the top, there will remain the solid content of the *frustum* of the cone.

How the content of the entire cone may be found, appears thus: Let ABCD be the *frustum* of the cone (either right or scalenous as the figures 2. and 3.): Let the cone ECD be supposed to be compleat: Let AG be drawn parallel to DE, and AH and EF be perpendicular on CD; it will be (by 2d 6. Eucl.) as $CG : CA :: CD : CE$; but (by the 4th Prop. of the same book) as $CA : AH :: CE : EF$;

CE : EF ; consequently (by 22d 5. Eucl.) as CG : AH :: CD : EF ; that is, as the excess of the diameter of the lesser base is to the height of the *frustum*, so is the diameter of the greater base to the height of the entire cone.

COROLLARY. FIG. 4.

Some casks whose staves are remarkably bended about the middle, and straight towards the ends, may be taken for two portions of cones, without any considerable error. Thus ABEF is a *frustum* of a right cone, to whose base EF, on the other side, there is another similar *frustum* of a cone joined EDCF. The vertices of these cones, if they be supposed to be compleated, will be found at G and H. Whence, by the preceding Prop. the solid content of such vessels may be found.

PROP.

P R O P. VI.

P R O B. FIG. 5.

A CYLINDER circumscribed about a sphere, that is, having its base equal to a great circle of the sphere, and its height equal to the diameter of the sphere, is to the sphere as 3 to 2.

Let ABEC be the quadrant of a circle, and ABDC the circumscribed square; and likewise the triangle ADC; by the revolution of the figure about the right line AC, as axis, a hemisphere will be generated by the quadrant, a cylinder of the same base and height by the square, and a cone by the triangle. Let these three be cut any how by the plane HF, parallel to the base AB, the section in the cylinder will be a circle whose radius is FH, in the hemisphere a circle of the radius EF, and in the cone a circle of the radius GF.

By

By the 47th 1. Eucl. EAq , or $HFq = EFq$ and FAq taken together, (but $AFq = FGq$, because $AC = CD$); therefore the circle of the radius HF is equal to a circle of the radius EF , together with a circle of the radius GF ; and, since this is true every where, all the circles together described by the respective *radii* HF (that is the cylinder) are equal to all the circles described by the respective *radii* EF and FG (that is, to the hemisphere and the cone taken together); but, by 10th 12. Eucl. the cone generated by the triangle DAC is one third part of the cylinder generated by the square BC . Whence it follows, that the hemisphere generated by the rotation of the quadrant $ABEC$ is equal to the remaining two third parts of the cylinder, and that the whole sphere is $\frac{2}{3}$ of the double cylinder circumscribed about it.

This is that celebrated 39th Prop. 1. book of Archimedes of the sphere and cylinder, in which he determines the proportion of the cylinder to the sphere inscribed to be that of 3 to 2.

COROL-

C O R O L L A R Y I.

Hence it follows, that the sphere is equal to a cone whose height is equal to the semidiameter of the sphere, having for its base a circle equal to the superficies of the sphere, or to four great circles of the sphere, or to a circle whose radius is equal to the diameter of the sphere, by 14th Prop. 2d part of this. And indeed a sphere differs very little from the sum of an infinite number of cones that have their bases in the surface of the sphere, and their common vertex in the center of the sphere ; so that the superficies of the sphere, (of whose dimension see 14th Prop. 2d part of this) multiplied into the third part of the semidiameter, gives the solid content of the sphere.

P R O P.

PROP. VII.

PROB. FIG. 6.

To find the Solid Content of a Sector of the Sphere.

A Spherical Sector ABC (as appears by the Corol. of the preceding Prop.) is very little different from an infinite number of cones, having their bases in the superficies of the sphere BEC, and their common vertex in the center. Wherefore the spherical superficies BEC, being found (by 15. Prop. 2d part), and multiplied into the third part of AB the radius of the sphere, the product will give the solid content of the sector ABC.

COROLLARY.

It is evident how to find the solidity of a spherical segment less than a hemisphere, by subtracting the cone ABC from the sector already

already found. But, if the spherical segment be greater than a hemisphere, the cone corresponding must be added to the sector, to make the segment.

P R O P. VIII.

P R O B. FIG. 7.

To find the solidity of the spheroid, and of its segments cut by planes perpendicular to the axis.

IN the 2d Prop. of this part it is shown, that every where $EH : EG :: CF : CD$; but circles are as the squares described upon their rays; that is, the circle of the radius EH is to the circle of the radius EG , as CFq to CDq . And since it is so every where, all the circles described with the respective rays EH , (that is, the spheroid made by the rotation of the semi-ellipsis AFB around the axis AB) will be to all the circles described

Q

by

by the respective *radii* EG, (that is, the sphere described by the rotation of the semicircle ADB on the axis AB) as FCq to CDq; that is, as the spheroid to the sphere on the same axis, so is the square of the other axis of the generating ellipse to the square of the axis of the sphere.

And this holds, whether the spheroid be found by a revolution around the greater or lesser axis.

C O R O L L A R Y 1.

Hence it appears, that the half of the spheroid, formed by the rotation of the space AHFC around the axis AC, is double of the cone generated by the triangle AFC about the same axis; which is the 32d Prop. of Archimedes, of conoids and spheroids.

C O R O L L A R Y 2.

Hence likewise is evident the measure of segments of the spheroid cut by planes
perpen-

perpendicular to the axis. For the segment of the spheroid made by the rotation of the space ANHE, round the axis AE, is to the segment of the sphere having the same axis AC, and made by the rotation of the segment of the circle AMGE, as CFq to CDq.

But, if the measure of this solid be wanted with less labour by the 34th Prop. of Archimedes, of conoids and spheroids, it will be as BE to $AC + EB$, so is the cone generated by the rotation of the triangle AHE round the axis AE, to the segment of the sphere made by the rotation of the space ANHE round the same axis AE; which could easily be demonstrated (was this a proper place for it) by the method of indivisibles.

C O R O L L A R Y 3.

Hence it is easy to find the solid content of the segment of a sphere or spheroid intercepted between two parallel planes, perpendicular to the axis. This agrees as well

to

to the oblate as to the oblong spheroid ; as is obvious.

C O R O L L A R Y 4. FIG. 8.

If a cask is to be valued as the middle piece of an oblong spheroid, cut by the two planes DC and FG, at right angles to the axis: First, Let the solid content of the half spheroid ABCED be measured by the preceding Prop. from which let the solidity of the segment DEC be subtracted, and there will remain the segment ABCD ; and this doubled will give the capacity of the cask required

The following method is generally made use of for finding the solid content of such vessels. The double area of the greatest circle, that is, of that which is described by the diameter AB at the middle of the cask, is added to the area of the circle at the end, that is, of the circle DC or FG (for they are usually equal), and the third part of this sum is taken for a mean base of the cask ; which
therefore

therefore multiplied into the length of the cask OP, gives the content of the vessel required.

Sometimes vessels have other figures different from those we have mentioned; the easy methods of measuring which may be learned from those who practise this art. What hath already been delivered is sufficient for our purpose.

P R O P. IX.

PROB. FIG. 9. and 10.

To find how much is contained in a vessel that is in part empty, whose axis is parallel to the horizon.

LET AGBH be the great circle in the middle of the cask, whose segment GBH is filled with liquor, the segment GAH being empty; the segment GBH is known, if the depth EB be known, and EH

a mean proportional between the segments of the diameter AB and EB; which are found by a rod or ruler put into the vessel at the orifice. Let the basis of the cask, at a medium, be found, which suppose to be the circle CKDL; and let the segment KCL be similar to the segment GAH (which is either found by the rule of three, because, as the circle AGBH is to the circle CKDL, so is the segment GAH to the segment KCL; or is found from the tables of segments made by authors); and the product of this segment multiplied by the length of the cask will give the liquid content remaining in the cask.

P R O P. X. P R O B.

To find the solid content of a regular and ordinate body.

A Tetraedron being a pyramid, the solid content is found by the 2d Prop. of this part. The Hexaedron, or cube being

a

a kind of prism, it is measured by the 1st Prop. of this part. An Octaedron consists of two pyramids of the same square base and of equal heights; consequently its measure is found from the 2d Prop. of this part. A Dodecaedron consists of twelve pyramids having equal equilateral and equiangular pentagonal bases; and so one of these being measured (by 2d Prop. of this) and multiplied by 12, the product will be equal to the solid content of the Dodecaedron. The Icosiaedron consists of 20 equal pyramids having triangular bases; the solid content of one of which being found (by the 2d Prop. of this) and multiplied by 20, gives the whole solid. The bases and heights of these pyramids, if you want to proceed more exactly, may be found by Trigonometry.

PROP.

PROP. XI. PROB.

To find the solid content of a body, however irregular.

LET the given body be immersed into a vessel of water, having the figure of a parallelopipedon or prism, and let it be noted how much the water is raised upon the immersion of the body. For it is plain that the space which the water fills, after the immersion of the body, exceeds the space filled before its immersion, by a space equal to the solid content of the body, however irregular. But, when this excess is of the figure of a parallelopipedon or prism, it is easily measured by the first Prop. of this part, to wit, by multiplying the area of the base, or mouth of the vessel, into the difference of the elevations of the water before and after immersion. Whence is found the solid content of the body given. *Q. E. I.*
In

In the same way the solid content of a part of a body may be found, by immersing that part only in water.

There is no necessity to insist here on diminishing or enlarging solid bodies in a given proportion. It will be easy to deduce these things from the 11th and 12th books of Euclid.

‘ The following rules are subjoined for
‘ the ready computation of the contents of
‘ vessels, and of any solids, in the measures
‘ in use in Great Britain.

‘ I. To find the content of a cylindric
‘ vessel in English wine-gallons, the dia-
‘ meter of the base and altitude of the vessel
‘ being given in inches and decimals of an
‘ inch.

‘ Square the number of inches in the di-
‘ ameter of the vessel ; multiply this square
‘ by the number of inches in the height :
‘ Then multiply the product by the decimal
‘ fraction .0034 ; and this last product shall
‘ give the content in wine-gallons and de-
‘ cimals of such a gallon. To express the

R

‘ rule

' rule arithmetically. Let D represent the
 ' number of inches and decimals of an inch
 ' in the diameter of the vessel, and H the
 ' inches and decimals of an inch in the
 ' height of the vessel; then the content in
 ' wine-gallons shall be $DDH \times \frac{34}{10000}$, or
 ' $DDH \times .0034$. *Ex.* Let the diameter
 ' $D = 51.2$ inches, the height $H = 62.3$ inches,
 ' then the content shall be $51.2 \times 51.2 \times 62.3$
 ' $\times .0034 = 555.27,342$ wine-gallons. This
 ' rule follows from Prop. 7. of the second
 ' part, and Prop. 3. of the third part; for, by
 ' the former, the area of the base of the ves-
 ' sels is in square inches $DD \times 7854$; and, by
 ' the latter, the content of the vessel in solid
 ' inches is $DDH \times .7854$; which divided by
 ' 231 (the number of cubical inches in a
 ' wine-gallon) gives $DDH \times .0034$, the con-
 ' tent in wine-gallons. But, though the
 ' charges in the excise are made (by statute)
 ' on the supposition that the wine-gallon
 ' contains 231 cubical inches; yet it is said,
 ' that, in sale, 224 cubical inches, the con-
 ' tent of the standard measured in Guildhall
 ' (as

‘ (as was mentioned above) are allowed to
 ‘ be a wine-gallon.

‘ II. Supposing the English ale-gallon to
 ‘ contain 282 cubical inches, the content of
 ‘ a cylindric vessel is computed, in such gal-
 ‘ lons, by multiplying the square of the di-
 ‘ ameter of a vessel by its height, as former-
 ‘ ly, and their product by the decimal frac-
 ‘ tion .0,027,851. That is, the solid content
 ‘ in ale-gallons is $DDH \times .0,027.851$.

‘ III. Supposing the Scots pint to contain
 ‘ about 103.4 cubical inches, (which is the
 ‘ measure given by the standards at Edin-
 ‘ burgh, according to experiments mention-
 ‘ ed above), the content of a cylindric ves-
 ‘ sel is computed in Scots pints, by multi-
 ‘ plying the square of the diameter of the
 ‘ vessel by its height, and the product of
 ‘ these by the decimal fraction .0076. Or
 ‘ the content of such a vessel in Scots pints
 ‘ is $DDH \times .0076$.

‘ IV. Supposing the Winchester bushel
 ‘ to contain 2187 cubical inches, the con-
 ‘ tent of a cylindric vessel is computed in
 ‘ those

' those bushels by multiplying the square of
 ' the diameter of the vessel by the height,
 ' and the product by the decimal fraction
 ' .0,003,606. But the standard bushel ha-
 ' ving been measured by Mr Everard and
 ' others in 1696, it was found to contain
 ' only 2145.6 solid inches; and therefore it
 ' was enacted, in the act for laying a duty
 ' upon malt, *That every round bushel, with*
 ' *a plain and even bottom, being $18\frac{1}{2}$ inches*
 ' *diameter throughout, and 8 inches deep,*
 ' *should be esteemed a legal Winchester bu-*
 ' *shel.* According to this act (ratified in the
 ' first year of Queen Anne) the legal Win-
 ' chester bushel contains only 2150.42 so-
 ' lid inches. And the content of a cylin-
 ' dric vessel is computed in such bushels, by
 ' multiplying the square of the diameter by
 ' the height, and their product by the de-
 ' cimal fraction .0,003,625. Or the con-
 ' tent of the vessel in those bushels is
 ' $DDH \times .0,003,625$.

' V. Supposing the Scots wheat firlo to
 ' contain $21\frac{1}{2}$ Scots pints, (as is appointed
 ' by

' by the statute 1618), and the pint to be
 ' conform to the Edinburgh standards above
 ' mentioned, the content of a cylindric vessel
 ' in such firlots is computed by multiplying
 ' the square of the diameter by the height,
 ' and their product by the decimal fraction
 ' .00,358. This firlot, in 1426, is appoint-
 ' ed to contain 17 pints; in 1457, it was ap-
 ' pointed to contain 18 pints; in 1587, it is
 ' $19\frac{1}{4}$ pints; in 1628, it is $21\frac{1}{4}$ pints: And
 ' though this last statute appears to have
 ' been founded on wrong computations in
 ' several respects; yet this part of the act
 ' that relates to the number of pints in the
 ' firlot seems to be the least exceptionable;
 ' and therefore we suppose the firlot to con-
 ' tain $21\frac{1}{4}$ pints of the Edinburgh standard,
 ' or about 2197 cubical inches; which a lit-
 ' tle exceeds the Winchester bushel, from
 ' which it may have been originally copied.

' VI. Supposing the bear-firLOT to contain
 ' 31 Scots pints, (according to the statute
 ' 1618), and the pint conform to the Edin-
 ' burgh standards, the content of a cylindric
 ' vessel

‘ vessel in such firlots is found by multiply-
 ‘ ing the square of the diameter by the
 ‘ height, and this product by .000,245.

‘ When the section of the vessel is not a
 ‘ circle, but an ellipsis, the product of the
 ‘ greatest diameter by the least, is to be
 ‘ substituted in those rules for the square of
 ‘ the diameter.

‘ VII. To compute the content of a ves-
 ‘ sel that may be considered as a *frustum* of
 ‘ a cone in any of those measures.

‘ Let A represent the number of inches
 ‘ in the diameter of the greater base, B the
 ‘ number of inches in the diameter of the
 ‘ lesser base. Compute the square of A, the
 ‘ product of A multiplied by B, and the
 ‘ square of B, and collect these into a sum.
 ‘ Then find the third part of this sum, and
 ‘ substitute it in the preceding rules in the
 ‘ place of the square of the diameter ; and
 ‘ proceed in all other respects as before.
 ‘ Thus, for example, the content in wine-
 ‘ gallons is $\overline{AA \times AB \times BB} \times \frac{1}{3} \times H \times$
 ‘ .0034.

‘ Or,

' Or, to the square of half the sum of the
 ' diameters A and B, add one third part of
 ' the square of half their difference, and
 ' substitute this sum in the preceding rules
 ' for the square of the diameter of the
 ' vessel ; for the square of $\frac{1}{2} A \times \frac{1}{2} B$ added
 ' to $\frac{1}{3}$ of the square of $\frac{1}{2} A - \frac{1}{2} B$, gives $\frac{1}{3}$
 ' $AA \times \frac{1}{2} AB \times \frac{1}{3} BB$.

' VIII. When a vessel is a *frustum* of a
 ' parabolic conoid, measure the diameter of
 ' the section at the middle of the height of
 ' the *frustum* ; and the content will be
 ' precisely the same as of a cylinder of this
 ' diameter, of the same height with the
 ' vessel.

' IX. When a vessel is a *frustum* of a
 ' sphere, if you measure the diameter of the
 ' section at the middle of the height of the
 ' *frustum*, then compute the content of a
 ' cylinder of this diameter of the same
 ' height with the vessel, and from this sub-
 ' tract $\frac{1}{3}$ of the content of a cylinder of the
 ' same height, on a base whose diameter is
 ' equal to its height ; the remainder will
 ' give

‘ give the content of the vessel. That is, if
 ‘ D represent the diameter of the middle
 ‘ section, and H the height of the *frustum*,
 ‘ you are to substitute $DD - \frac{1}{3} HH$ for the
 ‘ square of the diameter of the cylindric
 ‘ vessel in the first six rules.

‘ X. When the vessel is a *frustum* of a
 ‘ spheroid, if the bases are equal, the con-
 ‘ tent is readily found by the rule in p. 104.
 ‘ In other cases, let the axis of the solid be
 ‘ to the conjugate axis, as n to 1; let D be
 ‘ the diameter of the middle section of the
 ‘ *frustum*, H the height or length of the
 ‘ *frustum*; and substitute in the first six
 ‘ rules $DD - \frac{HH}{3nn}$ for the square of the square
 ‘ of the diameter of the vessel.

‘ XI. When the vessel is an hyperbolic
 ‘ conoid, let the axis of the solid be to the
 ‘ conjugate axis, as n to 1, D the diameter of
 ‘ the section at the middle of the *frustum*, H
 ‘ the height or the length: Compute DD
 ‘ $\times \frac{1}{3nn} \times HHH$, and substitute this sum for
 ‘ the square of the diameter of the cylindric
 ‘ vessel in the first six rules.

‘ XII.

' XII. In general, it is usual to measure
 ' any round vessel, by distinguishing it into
 ' several *frustums*, and taking the diameter
 ' of the section at the middle of each *frustum*;
 ' thence to compute the content of each, as
 ' if it was a cylinder of that mean diameter ;
 ' and to give their sum as the content of
 ' the vessel. From the total content, com-
 ' puted in this manner, they subtract suc-
 ' cessively the numbers which express the
 ' circular areas that correspond to those
 ' mean diameters, each as often as there are
 ' inches in the altitude of the *frustum* to
 ' which it belongs, beginning with the up-
 ' permost ; and in this manner calculate a
 ' table for the vessel, by which it readily ap-
 ' pears how much liquor is at any time con-
 ' tained in it, by taking either the dry or
 ' wet inches ; having regard to the inclina-
 ' tion or drip of the vessel, when it has any.

' This method of computing the content
 ' of a *frustum* from the diameter of the sec-
 ' tion at the middle of its height, is exact
 ' in that case only when it is a portion of a

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' parabolic

' parabolic conoid ; but in such vessels as are
 ' in common use, the error is not consider-
 ' able. When the vessel is a portion of a
 ' cone or hyperbolic conoid, the content, by
 ' this method, is found less than the truth ;
 ' but, when it is a portion of a sphere or
 ' spheroid, the content computed in this
 ' manner exceeds the truth. The difference
 ' or error is always the same, in the dif-
 ' ferent parts of the same or of similar ves-
 ' sels, when the altitude of the *frustum* is
 ' given. And when the altitudes are diffe-
 ' rent, the error is in the triplicate *ratio* of
 ' the altitude. If exactness be required, the
 ' error in measuring the *frustum* of a coni-
 ' cal vessel, in this manner, is $\frac{1}{4}$ of the con-
 ' tent of a cone similar to the vessel, of an
 ' altitude equal to the height of the *frustum*.
 ' In a sphere, it is $\frac{1}{3}$ of a cylinder, of a dia-
 ' meter and height equal to the *frustum*. In
 ' the spheroid and hyperbolic conoid, it is
 ' the same as in a cone generated by the
 ' right angled triangle, contained by the two
 ' semiaxes of the figure, revolving about
 ' that

‘ that side which is the femiaxis of the
 ‘ *frustum*. These are demonstrated in a treatise
 ‘ of fluxions by Mr Colin M’Laurin, p.
 ‘ 22. and 715. where those theorems are
 ‘ bounded by planes oblique to the axis in
 ‘ all the solids that are generated by any
 ‘ conic section revolving about either axis.

‘ In the usual method of computing a
 ‘ table for a vessel, by subducting from the
 ‘ whole content the number that expresses
 ‘ the uppermost area, as often as there are
 ‘ inches in the uppermost *frustum*, and afterwards
 ‘ the numbers for the other areas
 ‘ successively; it is obvious that the contents
 ‘ assigned by the table, when a few of the
 ‘ uppermost inches are dry, are stated a little
 ‘ too high, if the vessel stands on its lesser
 ‘ base, but too low when it stands on
 ‘ its greater base; because, when one inch
 ‘ is dry, for example, it is not the area at
 ‘ the middle of the uppermost *frustum*, but
 ‘ rather the area at the middle of the upper-
 ‘ most inch, that ought to be subducted
 ‘ from the total content, in order to find
 ‘ the content in this case.

‘ XIII. To measure round timber, let
 ‘ the mean circumference be found in feet
 ‘ and decimals of a foot ; square it ; multi-
 ‘ ply this square by the decimal .079,577,
 ‘ and the product by the length. Ex. Let
 ‘ the mean circumference of a tree be 10.3
 ‘ feet, and the length 24 feet. Then $10.3 \times$
 ‘ $10.3 \times 0.079,577 \times 24 = 202.615$, is the
 ‘ number of cubical feet in the tree. The
 ‘ foundation of this rule is, that, when the
 ‘ circumference of a circle is 1, the area is
 ‘ .0,795,774,715, and that the areas of
 ‘ circles are as the squares of their circum-
 ‘ ferences.

‘ But the common way used by artificers
 ‘ for measuring round timber, differs much
 ‘ from this rule. They call one fourth
 ‘ part of the circumference the *girt*, which
 ‘ is by them reckoned the side of a square,
 ‘ whose area is equal to the area of the sec-
 ‘ tion of the tree ; therefore they square
 ‘ the *girt*, and then multiply by the length
 ‘ of the tree. According to their method,
 ‘ the tree of the last example would be com-
 ‘ puted at 159.13 cubical feet only.

‘ How

‘ How square timber is measured will
 ‘ be easily understood from the preceding
 ‘ Propositions. Fifty solid feet of hewn
 ‘ timber, and forty of rough timber, make
 ‘ a load.

‘ XIV. To find the burden of a ship, or
 ‘ the number of tons it will carry, the fol-
 ‘ lowing rule is commonly given. Multi-
 ‘ ply the length of the keel taken within
 ‘ board, by the breadth of the ship within
 ‘ board, taken from the mid-ship beam from
 ‘ plank to plank, and the product by the
 ‘ depth of the hold, taken from the plank
 ‘ below the keelson to the under part of the
 ‘ upper deck plank, and divide the product
 ‘ by 94, the quotient is the content of the
 ‘ tonnage required. This rule, however,
 ‘ cannot be accurate ; nor can one rule be
 ‘ supposed to serve for the measuring exact-
 ‘ ly the burden of ships of all sorts. Of this
 ‘ the reader will find more in the Memoirs
 ‘ of the Royal Academy of Sciences at Paris
 ‘ in the year 1721.

‘ Our

' Our author having said nothing of
 ' weights, it may be of use to add briefly,
 ' that the English Troy-pound contains 12
 ' ounces, the ounce 20 penny weight, and
 ' the penny weight 24 grains; that the A-
 ' verdupois pound contains 16 ounces, the
 ' ounce 16 drams, and that 112 pounds is
 ' usually called the hundred weight. It is
 ' commonly supposed that 14 pounds Aver-
 ' dupois are equal to 17 pounds Troy. Ac-
 ' cording to Mr Everard's experiments, one
 ' pound Averdupois is equal to 14 ounces
 ' 11 penny-weight and 16 grains Troy,
 ' that is, to 7000 grains; and an Averdu-
 ' pois ounce is $437\frac{1}{2}$ grains. The Scots
 ' Troy-pound (which, by the statute 1718,
 ' was to be the same with the French) is
 ' commonly supposed equal to $15\frac{3}{4}$ ounces
 ' English Troy, or 7560 grains. By a mean
 ' of standards kept by the Dean of Guild
 ' of Edinburgh, it is $7599\frac{3}{4}$ or 7600 grains.
 ' They who have measured the weights
 ' which were sent from London, after the
 ' union of the kingdoms, to be the standards
 ' by

' by which the weights in Scotland should
 ' be made, have found the English Averdu-
 ' pois pound (from a medium of the several
 ' weights) to weigh 7000 grains, the same
 ' as Mr Everard ; according to which, the
 ' Scots, Paris, or Amsterdam pound, will
 ' be to the pound Averdupois as 38 to 35.
 ' The Scots Troy-stone contains 16 pounds,
 ' the pound two marks or 16 ounces, an
 ' ounce 16 drops, a drop 36 grains. Twen-
 ' ty Scots ounces make a Tron pound ;
 ' but, because it is usual to allow one to
 ' the score, the Tron pound is commonly
 ' 21 ounces. Sir John Skene, however,
 ' makes the Tron stone to contain only
 ' $19\frac{1}{2}$ pounds.'

F I N I S.

